

# Multiobjective particle swarm optimization for environmental/economic dispatch problem

M.A. Abido

Electrical Engineering Department, King Fahd University of Petroleum & Minerals, Dhahran 31261, Saudi Arabia

## ARTICLE INFO

### Article history:

Received 27 March 2008  
 Received in revised form 7 November 2008  
 Accepted 4 February 2009  
 Available online 17 March 2009

### Keywords:

Environmental/economic dispatch  
 Particle swarm optimization  
 Multiobjective optimization  
 Nondominated solutions  
 Pareto-optimal front

## ABSTRACT

A new multiobjective particle swarm optimization (MOPSO) technique for environmental/economic dispatch (EED) problem is proposed in this paper. The proposed MOPSO technique evolves a multiobjective version of PSO by proposing redefinition of global best and local best individuals in multiobjective optimization domain. The proposed MOPSO technique has been implemented to solve the EED problem with competing and non-commensurable cost and emission objectives. Several optimization runs of the proposed approach have been carried out on a standard test system. The results demonstrate the capabilities of the proposed MOPSO technique to generate a set of well-distributed Pareto-optimal solutions in one single run. The comparison with the different reported techniques demonstrates the superiority of the proposed MOPSO in terms of the diversity of the Pareto-optimal solutions obtained. In addition, a quality measure to Pareto-optimal solutions has been implemented where the results confirm the potential of the proposed MOPSO technique to solve the multiobjective EED problem and produce high quality nondominated solutions.

© 2009 Elsevier B.V. All rights reserved.

## 1. Introduction

The increasing public awareness of the environmental protection has forced the utilities to modify their design or operational strategies to reduce pollution and environmental emissions of the thermal power plants [1–3]. The emission dispatching option is an attractive short-term alternative in which both emission and fuel cost is to be minimized. In recent years, this option has received much attention [4–8] since it requires only small modification of the basic economic dispatch to include emissions.

Several techniques to handle the environmental/economic dispatch (EED) problem have been reported [4–13]. Generally speaking, there are three approaches to solve EED problem. The *first* approach treats the emission as a constraint with a permissible limit [4]. This formulation, however, has a severe difficulty in getting the trade-off relations between cost and emission.

The *second* approach treats the emission as another objective in addition to usual cost objective [5–8]. However, the EED problem was converted to a single objective problem either by linear combination of both objectives or by considering one objective at a time for optimization. Unfortunately, this approach requires multiple runs as many times as the number of desired Pareto-optimal solutions and tends to find weakly nondominated solutions.

The *third* approach handles both fuel cost and emission simultaneously as competing objectives. Stochastic search and fuzzy-based multiobjective optimization techniques have been proposed for the EED problem [9–11]. However, the algorithms do not provide a systematic framework for directing the search towards Pareto-optimal front and the extension of these techniques to include more objectives is a very involved question. In addition, these techniques are computationally involved and time-consuming. Genetic algorithm-based multiobjective techniques have been presented in [12,13] where multiple nondominated solutions can be obtained in a single run. However, genetic algorithm-based techniques suffer from premature convergence and the technique presented in [12] is computationally involved due to ranking process during the fitness assignment procedure.

Unlike genetic algorithm and other heuristic techniques, particle swarm optimization (PSO) has a flexible and well-balanced mechanism to enhance and adapt the global and local exploration abilities. It usually results in faster convergence rates than the genetic algorithm [14]. In recent years, PSO has been successfully implemented to different power system optimization problems including the economic power dispatch problem with impressive success [15–17]. The potential of PSO to handle nonsmooth and nonconvex economic power dispatch problem was demonstrated and reported [16,17]. However, the problem was formulated as a conventional dispatch problem with the fuel cost as the only objective considered for optimization.

Generally, changing conventional single objective PSO to a multiobjective PSO requires redefinition of global and local best

E-mail address: [mabido@kfupm.edu.sa](mailto:mabido@kfupm.edu.sa).

individuals in order to obtain a front of optimal solutions. In multi-objective particle swarm optimization, there is no absolute global best, but rather a set of nondominated solutions. In addition, there may be no single local best individual for each particle of the swarm. Choosing the global best and local best to guide the swarm particles becomes nontrivial task in multiobjective domain.

A little effort has been recently reported to implement MOPSO for solving power system problems. Wang and Singh [18] presented a fuzzified MOPSO to solve EED problem with heat dispatch and with multiple renewable energy sources. The approach presents a fuzzification mechanism for the selection of global best individual with interpreting the global best as an area, not just as a point. On the other hand, only one local best solution is maintained for each particle. This will degrade the search capability and violates the principle of multiobjective optimization. Kitamura et al. [19] presented a modified MOPSO to optimize an energy management system where the problem is solved in three phases by dividing the original optimization problem into partial problems. However, this approach has severe limitation in the case of strong interaction among the constraints in different subproblems. Hazra and Sinha [20] presented a MOPSO based approach to solve the congestion management problem where the cost and congestion are simultaneously minimized. In this approach the sigma method [21] is adopted to find the best local guide for a particle. However, the use of the sigma values increases the selection pressure of PSO which is already high. This may cause premature convergence in some cases, e.g., in multifrontal problems. A vector evaluated PSO (VEPSO) was proposed and examined for determining generator contributions to transmission system [22]. In VEPSO, fractions of the next generation are selected from the old generation according to each of the objectives separately. However, selection of individuals that excel in one objective without looking to the other objectives implies a problem of killing the middling performance individuals that can be very useful for compromise solutions [23].

In this paper, a novel multiobjective particle swarm optimization (MOPSO) technique is proposed and implemented for solving the environmental/economic dispatch problem. The proposed approach extends the single objective PSO by proposing new definitions of the local best and global best individuals in multiobjective optimization problems. Like other multiobjective evolutionary algorithms, a hierarchical clustering technique is implemented to manage Pareto-optimal set size and a fuzzy-based mechanism is employed to extract the best compromise solution. Several runs have been carried out on the standard IEEE 30-bus test system and the results are compared to different techniques reported in literature. The effectiveness and potential of the proposed MOPSO approach to solve the multiobjective EED problem are demonstrated.

## 2. Problem statement

The EED problem is to minimize two competing objective functions, fuel cost and emission, while satisfying several equality and inequality constraints. Generally the problem is formulated as follows.

### 2.1. Problem objectives

#### 2.1.1. Minimization of fuel cost

The total US\$/h fuel cost  $F(P_G)$  can be expressed as

$$F(P_G) = \sum_{i=1}^N a_i + b_i P_{G_i} + c_i P_{G_i}^2 \quad (1)$$

where  $N$  is the number of generators,  $a_i$ ,  $b_i$ , and  $c_i$  are the cost coefficients of the  $i$ th generator, and  $P_{G_i}$  is the real power output of the

$i$ th generator.  $P_G$  is the vector of real power outputs of generators and defined as

$$P_G = [P_{G_1}, P_{G_2}, \dots, P_{G_N}]^T \quad (2)$$

#### 2.1.2. Minimization of emission

The environmental pollutants such as sulphur oxides  $SO_x$  and nitrogen oxides  $NO_x$  caused by fossil-fueled thermal units can be modeled separately. However, for comparison purposes, the total ton/h emission  $E(P_G)$  of these pollutants can be expressed as [5,8]

$$E(P_G) = \sum_{i=1}^N 10^{-2} (\alpha_i + \beta_i P_{G_i} + \gamma_i P_{G_i}^2) + \zeta_i \exp(\lambda_i P_{G_i}) \quad (3)$$

where  $\alpha_i$ ,  $\beta_i$ ,  $\gamma_i$ ,  $\zeta_i$ , and  $\lambda_i$  are coefficients of the  $i$ th generator emission characteristics.

### 2.2. Problem constraints

#### 2.2.1. Generation capacity constraint

For stable operation, real power output of each generator is restricted by lower and upper limits as follows:

$$P_{G_i}^{\min} \leq P_{G_i} \leq P_{G_i}^{\max}, \quad i = 1, \dots, N \quad (4)$$

#### 2.2.2. Power balance constraints

The total power generation must cover the total demand  $P_D$  and the real power loss in transmission lines  $P_{loss}$ . Hence,

$$\sum_{i=1}^N P_{G_i} - P_D - P_{loss} = 0 \quad (5)$$

As a matter of fact, the power loss in transmission lines can be calculated by different methods such as B matrix loss formula method and power flow method. The second method has been adopted in our implementation where calculation of  $P_{loss}$  implies solving the load flow problem with equality constraints on real and reactive power at each bus as follows.

$$P_{G_i} - P_{D_i} - V_i \sum_{j=1}^{NB} V_j [G_{ij} \cos(\delta_i - \delta_j) + B_{ij} \sin(\delta_i - \delta_j)] = 0 \quad (6)$$

$$Q_{G_i} - Q_{D_i} - V_i \sum_{j=1}^{NB} V_j [G_{ij} \sin(\delta_i - \delta_j) - B_{ij} \cos(\delta_i - \delta_j)] = 0 \quad (7)$$

where  $i = 1, 2, \dots, NB$ ;  $NB$  is the number of buses;  $Q_G$  and  $Q_D$  are the generator and demand reactive power respectively;  $G_{ij}$  and  $B_{ij}$  are the transfer conductance and susceptance between bus  $i$  and bus  $j$  respectively;  $V_i$  and  $V_j$  are the voltage magnitudes at bus  $i$  and bus  $j$  respectively;  $\delta_i$  and  $\delta_j$  are the voltage angles at bus  $i$  and bus  $j$  respectively.

Then, the real power loss in transmission lines can be calculated as

$$P_{loss} = \sum_{k=1}^{NL} g_k [V_i^2 + V_j^2 - 2V_i V_j \cos(\delta_i - \delta_j)], \quad (8)$$

where  $NL$  is the number of transmission lines;  $g_k$  is the conductance of the  $k$ th line that connects bus  $i$  to bus  $j$ .

#### 2.2.3. Security constraints

For secure operation, the transmission line loading  $S_l$  is restricted by its upper limit as:

$$S_l \leq S_l^{\max}, \quad i = 1, \dots, NL \quad (9)$$

### 2.3. Problem formulation

Aggregating the objectives and constraints, the problem can be mathematically formulated as a nonlinear constrained multiobjective optimization problem as follows.

$$\text{Minimize } [F(P_G), E(P_G)]_{P_G} \quad (10)$$

subject to:

$$g(P_G) = 0 \quad (11)$$

$$h(P_G) \leq 0 \quad (12)$$

where  $g$  and  $h$  are the equality and inequality problem constraints respectively.

### 3. Principle of multiobjective optimization

Many real-world problems involve simultaneous optimization of several objective functions. Generally, these functions are non-commensurable and often competing and conflicting objectives. Multiobjective optimization with such conflicting objective functions gives rise to a set of optimal solutions, instead of one optimal solution. The reason for the optimality of many solutions is that no one can be considered to be better than any other with respect to all objective functions. These optimal solutions are known as *Pareto-optimal* solutions [24].

For a multiobjective optimization problem, any two solutions  $x_1$  and  $x_2$  can have one of two possibilities: one dominates the other or none dominates the other. In a minimization problem, without loss of generality, a solution  $x_1$  dominates  $x_2$  if the following two conditions are satisfied:

$$1. \quad \forall i \in \{1, 2, \dots, N_{obj}\} : f_i(x_1) \leq f_i(x_2) \quad (13)$$

$$2. \quad \exists j \in \{1, 2, \dots, N_{obj}\} : f_j(x_1) < f_j(x_2) \quad (14)$$

If any of the above conditions is violated, the solution  $x_1$  does not dominate the solution  $x_2$ . If  $x_1$  dominates the solution  $x_2$ ,  $x_1$  is called the nondominated solution. The solutions that are nondominated within the entire search space are denoted as *Pareto-optimal* and constitute the *Pareto-optimal set*. This set is also known as *Pareto-optimal front*.

### 4. The proposed MOPSO technique

#### 4.1. Overview

The recent studies on evolutionary algorithms have shown that the population-based algorithms are potential candidate to solve multiobjective optimization problems and can be efficiently used to eliminate most of the difficulties of classical single objective methods such as the sensitivity to the shape of the Pareto-optimal front and the necessity of multiple runs to find multiple Pareto-optimal solutions.

In general, the goal of a multiobjective optimization algorithm is not only to guide the search towards the Pareto-optimal front but also to maintain population diversity in the set of the Pareto-optimal solutions.

In recent years, PSO has been presented as an efficient population-based heuristic technique with a flexible and well-balanced mechanism to enhance and adapt the global and local exploration capabilities. However, changing conventional single objective PSO to a multiobjective PSO requires redefinition of global and local best individuals since, in multiobjective particle swarm optimization, there is no absolute global best, but rather a set of nondominated solutions. In addition, there may be no single local best individual for each particle of the swarm. Choosing the global

best and local best to guide the swarm particles becomes nontrivial task in multiobjective domain.

The proposed approach addresses the problem of evolving a multiobjective version of the conventional PSO where the global and local best individuals have been redefined and a mechanism for selection of these individuals has been proposed. It is worth mentioning that the proposed MOPSO technique has been implemented to several nontrivial standard test problems in multiobjective optimization domain with impressive success [25].

#### 4.2. Basic elements and definitions

The basic elements of the proposed MOPSO technique are briefly stated and defined as follows:

*Particle,  $X(t)$* : It is a candidate solution represented by an  $m$ -dimensional vector, where  $m$  is the number of optimized parameters. At time  $t$ , the  $j$ th particle  $X_j(t)$  can be described as  $X_j(t) = [x_{j,1}(t), \dots, x_{j,m}(t)]$ , where  $x_s$  are the optimized parameters and  $x_{j,k}(t)$  is the position of the  $j$ th particle with respect to the  $k$ th dimension, i.e., the value of the  $k$ th optimized parameter in the  $j$ th candidate solution.

*Population,  $pop(t)$* : It is a set of  $n$  particles at time  $t$ , i.e.,  $pop(t) = [X_1(t), \dots, X_n(t)]^T$ .

*Particle velocity,  $V(t)$* : It is the velocity of the moving particles represented by an  $m$ -dimensional vector. At time  $t$ , the  $j$ th particle velocity  $V_j(t)$  can be described as  $V_j(t) = [v_{j,1}(t), \dots, v_{j,m}(t)]$ , where  $v_{j,k}(t)$  is the velocity component of the  $j$ th particle with respect to the  $k$ th dimension. The particle velocity in the  $k$ th dimension is limited by some maximum value,  $v_{max}^k$ . This limit enhances the local exploration of the problem space. To ensure uniform velocity through all dimensions, the maximum velocity in the  $k$ th dimension is proposed as:

$$v_k^{max} = \frac{x_k^{max} - x_k^{min}}{N} \quad (15)$$

where  $N$  is a selected number of intervals.

*Inertia weight,  $w(t)$* : It is a control parameter that is used to control the impact of the previous velocities on the current velocity. Hence, it influences the trade-off between the global and local exploration abilities of the particles. For initial stages of the search process, large inertia weight to enhance the global exploration is recommended while, for last stages, the inertia weight is reduced for better local exploration. An annealing decrement function for decreasing the inertia weight given as  $w(t) = \alpha w(t-1)$ ,  $\alpha$  is a decrement constant smaller than but close to 1, is employed in this study.

*Nondominated local set,  $S_j^*(t)$* : It is a set that stores the nondominated solutions obtained by the  $j$ th particle up to the current time. As the  $j$ th particle moves through the search space, its new position is added to this set and the set is updated to keep only the nondominated solutions by applying the two dominance conditions given in Eqs. (13) and (14). An average linkage based hierarchical clustering algorithm [26] is employed to reduce the nondominated local set size if it exceeds a certain prespecified value. This clustering algorithm starts with initializing a distinct cluster for each individual. The average distance between pairs of individuals across all possible pairs of clusters is calculated. The two clusters with minimal distance are combined into a larger one. The procedure of joining the adjacent clusters will be iteratively repeated until the required number of groups is obtained.

*Nondominated global set,  $S^{**}(t)$* : It is a set that stores the nondominated solutions obtained by all particle up to the current time. First, the union of all nondominated local sets is formed. Then, the nondominated solutions out of this union are members in the nondominated global set. The clustering algorithm is employed to

reduce the nondominated global set to a prespecified manageable size.

**External set:** It is an archive that stores a historical record of the nondominated solutions obtained along the search process. This set is updated continuously after each iteration by applying the dominance conditions to the union of this set and the nondominated global set. Then, the nondominated solutions of this union are members in the updated external set. The clustering algorithm is also used to limit the external set to a prespecified manageable size.

**Local best,  $X_j^*(t)$ , and Global best,  $X_j^{**}(t)$ :** In order to guide the search towards the Pareto-optimal front, the global and local best individuals are selected as follows. The individual distances between members in nondominated local set of the  $j$ th particle,  $S_j^*(t)$ , and members in nondominated global set,  $S^{**}(t)$ , are measured in the objective space. If  $X_j^*(t)$  and  $X_j^{**}(t)$  are the members of  $S_j^*(t)$  and  $S^{**}(t)$  respectively that give the minimum distance, they are selected as the local best and the global best of the  $j$ th particle respectively.

#### 4.3. Computational flow

In the proposed MOPSO algorithm, the population has  $n$  particles and each particle is an  $m$ -dimensional vector, where  $m$  is the number of optimized parameters. The computational flow of the proposed MOPSO technique can be described in the following steps.

**Step 1 (Initialization):** Set the time counter  $t=0$  and generate randomly  $n$  particles,  $\{X_j(0), j=1, \dots, n\}$ , where  $X_j(0)=[x_{j,1}(0), \dots, x_{j,m}(0)]$ .  $x_{j,k}(0)$  is generated by randomly selecting a value with uniform probability over the  $k$ th optimized parameter search space  $[x_{\min}^k, x_{\max}^k]$ . Similarly, generate randomly initial velocities of all particles,  $\{V_j(0), j=1, \dots, n\}$ , where  $V_j(0)=[v_{j,1}(0), \dots, v_{j,m}(0)]$ .  $v_{j,k}(0)$  is generated by randomly selecting a value with uniform probability over the  $k$ th dimension  $[-v_{\max}^k, v_{\max}^k]$ . Each particle in the initial population is evaluated using the objective functions. For each particle, set  $S_j^*(0) = \{X_j(0)\}$  and the local best  $X_j^*(0) = X_j(0)$ ,  $j=1, \dots, n$ . Search for the nondominated solutions and form the nondominated global set  $S^{**}(0)$ . The nearest member in  $S^{**}(0)$  to  $X_j^*(0)$  is selected as the global best  $X_j^{**}(0)$  of the  $j$ th particle. Set the external set equal to  $S^{**}(0)$ . Set the initial value of the inertia weight  $w(0)$ .

**Step 2 (Time updating):** Update the time counter  $t=t+1$ .

**Step 3 (Weight updating):** Update the inertia weight  $w(t) = \alpha w(t-1)$ .

**Step 4 (Velocity updating):** Using the local best  $X_j^*(0)$  and the global best  $X_j^{**}(t)$  of each particle,  $j=1, \dots, n$ , the  $j$ th particle velocity in the  $k$ th dimension is updated according to the following equation:

$$v_{j,k}(t) = w(t)v_{j,k}(t-1) + c_1r_1(x_{j,k}^*(t-1) - x_{j,k}(t-1)) + c_2r_2(X_{j,k}^{**}(t-1) - x_{j,k}(t-1)) \quad (16)$$

where  $c_1$  and  $c_2$  are positive constants and  $r_1$  and  $r_2$  are uniformly distributed random numbers in  $[0,1]$ . If a particle violates the velocity limits, set its velocity equal to the proper limit.

**Step 5 (Position updating):** Based on the updated velocities, each particle changes its position according to the following equation

$$x_{j,k}(t) = v_{j,k}(t) + x_{j,k}(t-1) \quad (17)$$

If a particle violates its position limits in any dimension, set its position at the proper limit.

**Step 6 (Nondominated local set updating):** The updated position of the  $j$ th particle is added to  $S_j^*(t)$ . The dominated solutions in  $S_j^*(t)$  will be truncated and the set will be updated accordingly. If the

size of  $S_j^*(t)$  exceeds a prespecified value, the clustering algorithm will be invoked to reduce the size to its maximum limit.

**Step 7 (Nondominated global set updating):** The union of all nondominated local sets is formed and the nondominated solutions out of this union are extracted to be members in the nondominated global set  $S^{**}(t)$ . The size of this set will be reduced by clustering algorithm if it exceeds a prespecified value.

**Step 8 (External set updating):** The external Pareto-optimal set is updated as follows.

Copy the members of  $S^{**}(t)$  to the external Pareto set.

Search the external Pareto set for the nondominated individuals and remove all dominated solutions from the set.

If the number of the individuals externally stored in the Pareto set exceeds the maximum size, reduce the set by means of clustering.

**Step 9 (Local best and global best updating):** The individual distances between members in  $S_j^*(t)$ , and members in  $S^{**}(t)$ , are measured in the objective space. If  $X_j^*(t)$  and  $X_j^{**}(t)$  are the members of  $S_j^*(t)$  and  $S^{**}(t)$  respectively that give the minimum distance, they are selected as the local best and the global best of the  $j$ th particle respectively.

**Step 10 (Stopping criteria):** If the number of iterations exceeds its maximum preset limit then stop, else go to step 2.

Upon having the Pareto-optimal set of nondominated solution, fuzzy-based mechanism [6] to extract the best compromise solution is imposed to present one solution to the decision maker.

#### 4.4. Implementation

In this study, the proposed MOPSO technique has been developed in order to make it suitable for solving nonlinear constrained optimization problems. A procedure is imposed to check the feasibility of the candidate solutions in all stages of the search process. This ensures the feasibility of the nondominated solutions.

The proposed MOPSO technique has been implemented on a 3-GHz PC using FORTRAN language. In all optimization runs, the number of particles and the maximum number of generations were selected as 100 and 1000, respectively. The maximum size of the Pareto-optimal set was selected as 25 solutions while the local best set size is selected as 10 solutions. If the number of nondominated Pareto-optimal solutions in global best set and local best set exceeds the respective bound, the clustering technique is used.

The computational flow chart of the proposed MOPSO algorithm is depicted in Fig. 1.

### 5. Results and discussions

Having been applied for the first time, the proposed MOPSO technique was tested on the standard IEEE 30-bus 6-generator test system as several techniques have been tested on this standard system with reported results in the literature. The single-line diagram of the IEEE test system is shown in Fig. 2 and the detailed data are given in [5,8]. The values of fuel cost and emission coefficients are given in Table 1.

To demonstrate the effectiveness of the proposed approach, three cases with different complexity have been considered as follows:

- Case 1: The generation capacity and the power balance constraints with neglecting  $P_{loss}$  are considered.
- Case 2: The generation capacity and the power balance constraints with considering  $P_{loss}$  are considered.
- Case 3: All constraints discussed in the problem formulation are considered.

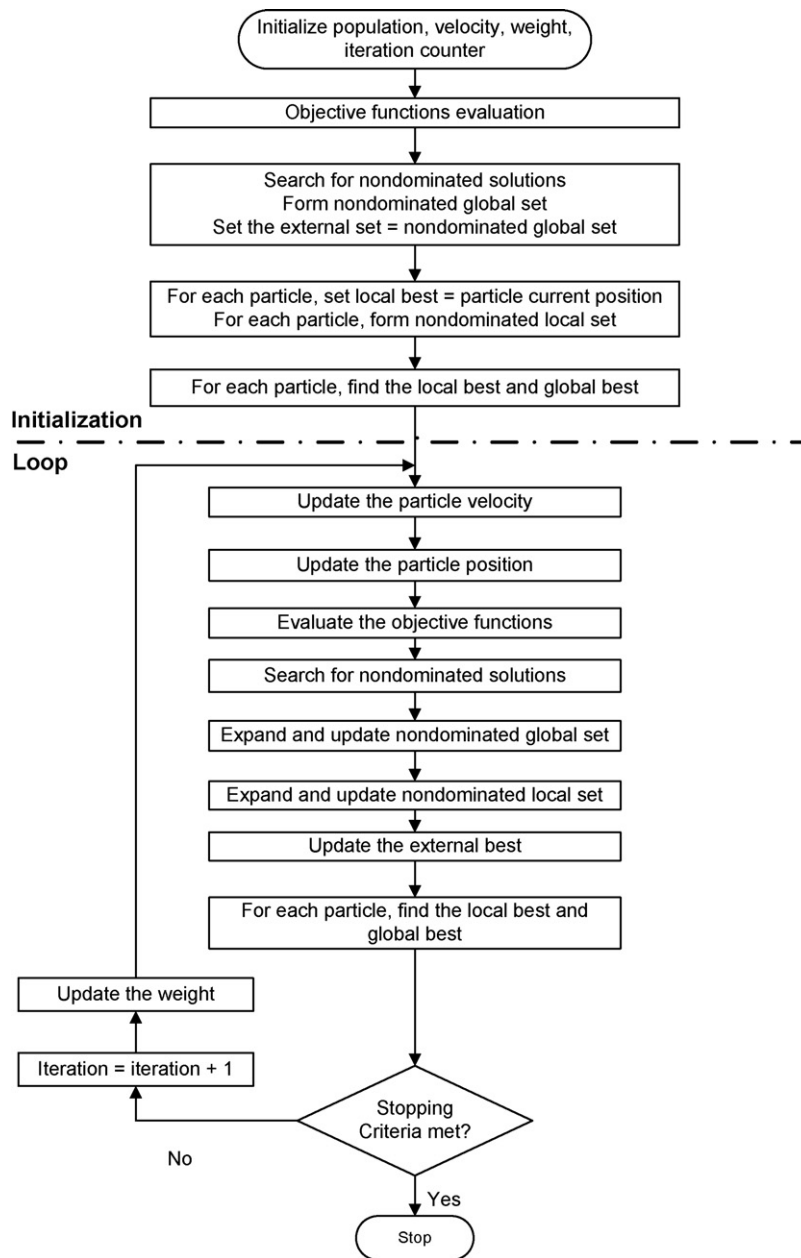


Fig. 1. Computational flow chart of the proposed MOPSO algorithm.

5.1. Single objective optimization

At first, fuel cost and emission objectives are optimized individually using single objective PSO in order to explore the extreme

Table 1  
Generator fuel cost and emission coefficients.

	$G_1$	$G_2$	$G_3$	$G_4$	$G_5$	$G_6$
Cost						
$a$	10	10	20	10	20	10
$b$	200	150	180	100	180	150
$c$	100	120	40	60	40	100
Emission						
$\alpha$	4.091	2.543	4.258	5.326	4.258	6.131
$\beta$	-5.554	-6.047	-5.094	-3.550	-5.094	-5.555
$\gamma$	6.490	5.638	4.586	3.380	4.586	5.151
$\zeta$	2.0E-4	5.0E-4	1.0E-6	2.0E-3	1.0E-6	1.0E-5
$\lambda$	2.857	3.333	8.000	2.000	8.000	6.667

points of the trade-off surface and evaluate the diversity characteristics of the Pareto-optimal solutions obtained by the proposed MOPSO technique. The best results of cost and emission functions when optimized individually using single objective PSO are given in Table 2. The PSO convergence with the individual objective optimization in all cases is shown in Fig. 3. The fast convergence of the PSO technique is quite evident as it takes only few iterations to reach the optimal solution.

5.2. Multiobjective optimization using the proposed MOPSO

The proposed MOPSO approach has been implemented to optimize cost and emission objectives simultaneously considering the three cases stated above. The distribution of the Pareto-optimal set over the trade-off surface is shown in Figs. 4–6 for the Cases 1, 2, and 3 respectively. It can be seen that the proposed MOPSO technique preserves the diversity of the nondominated solutions over the Pareto-optimal front and solve effectively the problem in all

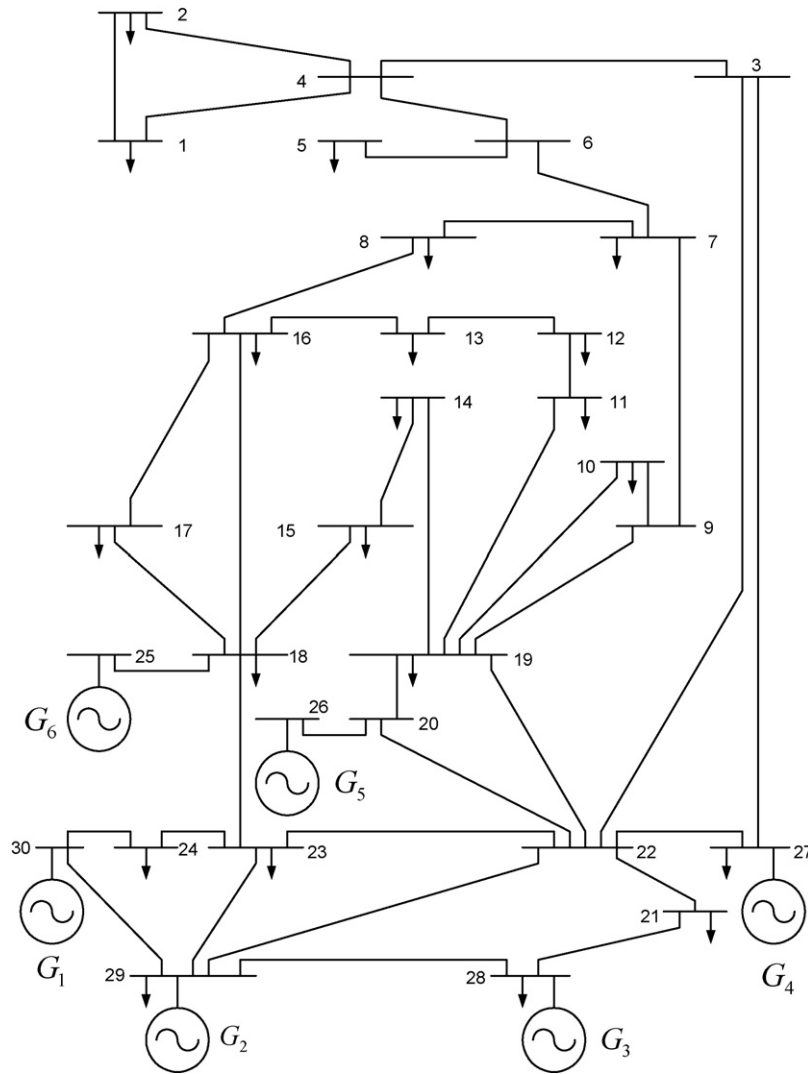


Fig. 2. Single-line diagram of IEEE 30-bus test system.

cases considered. It is worth mentioning that, in each case, the Pareto-optimal set has 25 nondominated solutions. Out of them, two nondominated solutions that represent the best cost and best emission are given in Table 3.

Comparing the results of single objective PSO given in Table 2 and those of the proposed MOPSO technique given in Table 3, it is clear that the results of the proposed MOPSO in all cases are almost identical with those of single objective PSO. This demonstrates the effectiveness of the proposed MOPSO to span over the entire Pareto-optimal front surface. In addition, the close agreement of the results shows clearly the capability of the proposed MOPSO technique to

handle multiobjective optimization problems as the best solution of each objective along with a manageable set of nondominated solutions can be obtained in one single run.

The results of the proposed approach are compared to those reported in the literature with Case 1 using linear programming (LP) [5], multiobjective stochastic search technique (MOSST) [11], and the recently developed strength Pareto evolutionary algorithm (SPEA) [13] where the results of these techniques are given in Table 4. Comparing the results of the proposed MOPSO given in Table 3 and those given in Table 4, it can be seen that the proposed MOPSO technique is superior compared to all reported techniques

Table 2  
The best solutions for cost and emission optimized individually.

	Case 1		Case 2		Case 3	
	Cost	Emission	Cost	Emission	Cost	Emission
$P_{G_1}$	0.1098	0.4061	0.1153	0.4104	0.1513	0.4569
$P_{G_2}$	0.2997	0.4590	0.3062	0.4629	0.3369	0.5121
$P_{G_3}$	0.5238	0.5377	0.5962	0.5436	0.7848	0.6518
$P_{G_4}$	1.0164	0.3833	0.9803	0.3896	1.0111	0.4335
$P_{G_5}$	0.5249	0.5379	0.5141	0.5437	0.1050	0.1990
$P_{G_6}$	0.3594	0.5099	0.3550	0.5149	0.4762	0.6142
Cost (US\$/h)	600.11	638.24	607.78	645.23	618.48	656.73
Emission (ton/h)	0.2221	0.1942	0.2198	0.1942	0.2302	0.2013

Table 3  
Best cost and best emission of the proposed MOPSO technique.

	Best cost			Best emission		
	Case 1	Case 2	Case 3	Case 1	Case 2	Case 3
$P_{G_1}$	0.1183	0.1207	0.1524	0.4015	0.4101	0.4589
$P_{G_2}$	0.3019	0.3131	0.3427	0.4590	0.4594	0.5121
$P_{G_3}$	0.5224	0.5907	0.7857	0.5332	0.5511	0.6524
$P_{G_4}$	1.0116	0.9769	1.0180	0.3891	0.3919	0.4331
$P_{G_5}$	0.5254	0.5155	0.0995	0.5456	0.5413	0.1981
$P_{G_6}$	0.3544	0.3504	0.4669	0.5057	0.5111	0.6129
Cost (US\$/h)	600.12	607.79	618.54	637.42	644.74	656.87
Emission (ton/h)	0.2216	0.2193	0.2308	0.1942	0.1942	0.2014

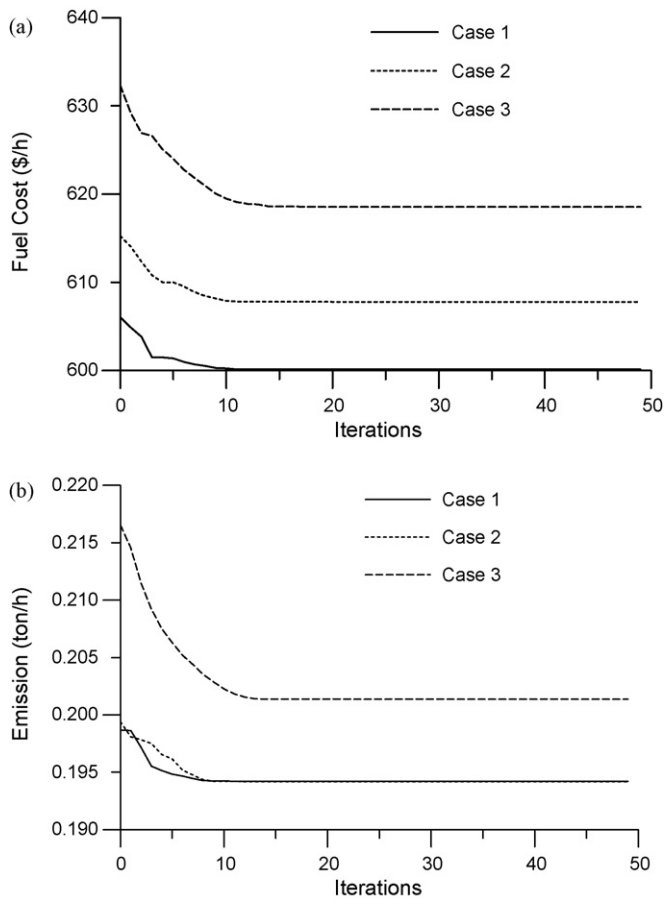


Fig. 3. Convergence of cost and emission objective functions in all cases. (a) Fuel cost convergence. (b) Emission convergence.

and gives better results in terms of the fuel cost saving achieved. It can be concluded that the proposed MOPSO is capable of exploring more efficient solutions. This demonstrates the potential and effectiveness of the proposed technique to solve EED problem.

The fuzzy-based mechanism [6] is used to evaluate each member of the Pareto-optimal set obtained by the proposed MOPSO technique. Then, the best compromise solution that has the maximum value of membership function can be extracted. This procedure has been applied in all cases and the best compromise solutions are given in Table 5. The best compromise solutions are also assigned in Figs. 4–6 for Cases 1, 2, and 3 respectively.

5.3. Robustness and quality measure of the proposed MOPSO technique

To demonstrate the effectiveness and robustness of the proposed MOPSO technique, 20 different optimization runs have been carried out in each case with different initializations. In addition, 20 different runs of SPEA implemented in [13] have been carried out in each case for comparison purposes. The best solutions obtained by each technique in all 20 runs are given in Table 6. It is quite evident that

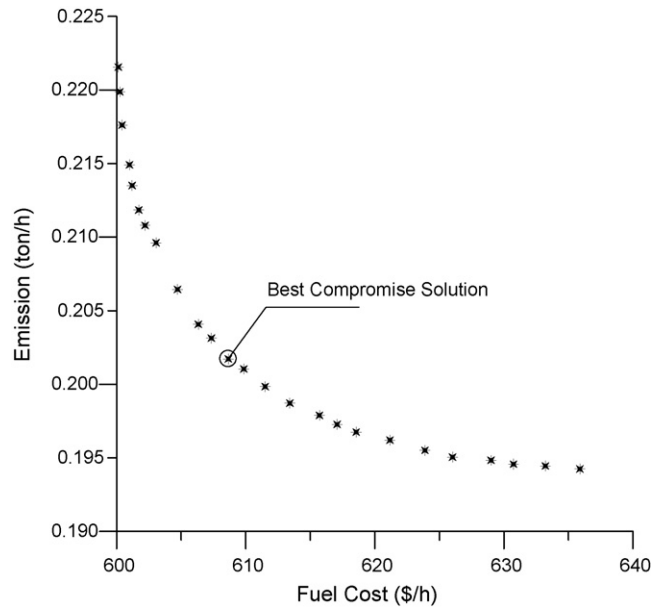


Fig. 4. Pareto-optimal front of the proposed approach in a single run, Case 1.

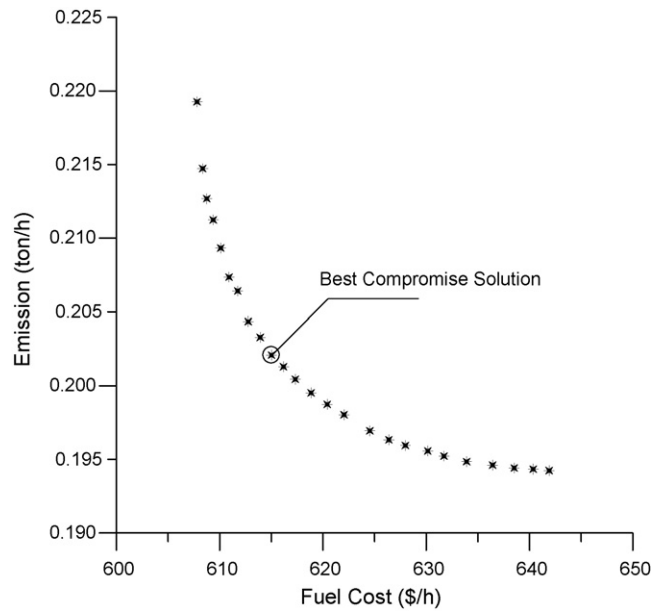


Fig. 5. Pareto-optimal front of the proposed approach in a single run, Case 2.

the proposed MOPSO gives better results in all cases compared to SPEA.

In this study, the quality of the Pareto-optimal solutions has been measured as follows. For 20 different optimization runs with 25 nondominated solutions per run, 500 nondominated solutions can be obtained. Combining the solutions obtained by the proposed MOPSO with those obtained by SPEA, a pool of 1000 nondominated

Table 4 Best cost and best emission of Case 1 of different techniques.

	Best cost			Best emission		
	LP [5]	MOSST [11]	SPEA [13]	LP [5]	MOSST [11]	SPEA [13]
Cost (US\$/h)	606.31	605.89	600.15	639.60	644.11	638.51
Emission (ton/h)	0.2233	0.2222	0.2215	0.1942	0.1942	0.1942

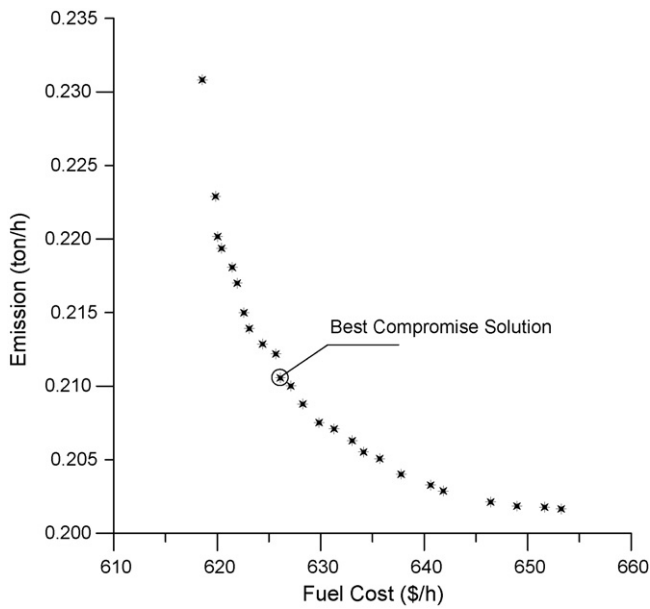


Fig. 6. Pareto-optimal front of the proposed approach in a single run, Case 3.

Table 5  
Best compromise solution of the proposed MOPSO technique.

	Case 1	Case 2	Case 3
$P_{G1}$	0.2516	0.2367	0.2882
$P_{G2}$	0.3770	0.3616	0.3965
$P_{G3}$	0.5283	0.5887	0.7320
$P_{G4}$	0.7124	0.7041	0.7520
$P_{G5}$	0.5566	0.5635	0.1489
$P_{G6}$	0.4081	0.4087	0.5463
Cost (US\$/h)	608.65	615.00	626.10
Emission (ton/h)	0.2017	0.2021	0.2106

solutions can be created. The quality measure presented in [27] has been implemented to the created pool where the dominance conditions have been applied to all solutions in the pool. The nondominated solutions are extracted from the pool to form an elite set of Pareto-optimal solutions obtained by both techniques. Generally, the larger the number of nondominated solutions in the elite set by a certain technique, the better the quality of the solutions obtained by this technique [27].

For each case, this quality measure has been employed. The number of the extracted nondominated solutions in the elite set of each case is 293, 283, and 157 for Cases 1, 2, and 3 respectively. The contributions of SPEA and the proposed MOPSO are given in Table 7. It can be observed that the proposed MOPSO has the majority of the elite set members in all cases considered. It can be concluded that the most of the nondominated solutions obtained by the proposed MOPSO are true Pareto-optimal solutions. This feature of the proposed MOPSO is more pronounced in Case 3 where approximately 72.6% of the elite set size is contributed by the proposed MOPSO technique. This observation is also confirmed in Fig. 7 that shows the Pareto-optimal fronts of both techniques where the superiority

Table 6  
The best solutions for cost and emission for 20 different optimization runs.

	Case 1		Case 2		Case 3	
	Cost (US\$/h)	Emission (ton/h)	Cost (US\$/h)	Emission (ton/h)	Cost (US\$/h)	Emission (ton/h)
SPEA	600.15	0.1942	607.81	0.1942	618.70	0.2016
MOPSO	600.12	0.1942	607.79	0.1942	618.54	0.2014

Table 7  
Quality measure results of nondominated solutions obtained by the proposed MOPSO approach.

	Number of Pareto solutions		Normalized distance	
	SPEA	MOPSO	SPEA	MOPSO
Case 1	146	147	0.9821	1.0000
Case 2	130	153	0.9791	1.0000
Case 3	43	114	0.9414	1.0000

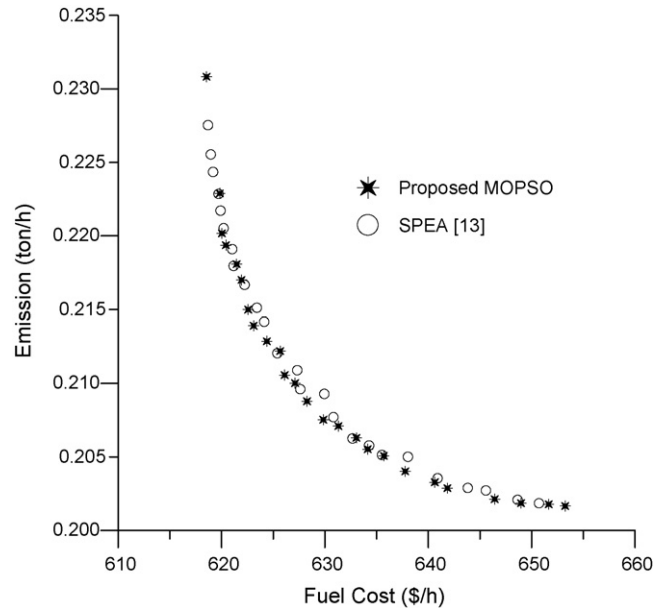


Fig. 7. Pareto-optimal fronts of the proposed MOPSO and SPEA [13], Case 3.

Table 8  
The average execution time.

	Case 1	Case 2	Case 3
SPEA	1.210	1.834	5.688
MOPSO	1.224	1.907	5.854

of the Pareto-optimal front of the proposed MOPSO is evident in terms of its diversity and quality.

In addition, the normalized distance between the outer nondominated solutions of each technique represented in the elite set is measured. The normalized distance results are given in Table 7. It can be seen that the nondominated solutions obtained by the proposed MOPSO span over the entire Pareto-optimal front in all cases as the extreme solutions are related to the proposed MOPSO technique since the normalized distance is 1.0 in all cases. It can be concluded that the proposed MOPSO has better diversity and quality characteristics of the nondominated solutions compared to SPEA for the problem under consideration.

The execution time for the proposed MOPSO is also assessed for the different cases considered. Table 8 shows the average execution time per generation of the proposed MOPSO which is slightly longer than that of SPEA. This can be attributed to the distance measurement between the global nondominated solutions and local nondominated solutions. However, this time can be reduced by reducing the size of the nondominated local set.

## 6. Conclusions

In this paper, a novel multiobjective particle swarm optimization technique has been proposed and applied to environmental/



economic power dispatch optimization problem. The proposed MOPSO technique presents a multiobjective version of the conventional PSO technique and utilizes its effectiveness to solve the multiobjective optimization problems. The EED problem has been formulated with competing fuel cost and environmental impact objectives. The results show the potential and efficiency of the proposed MOPSO technique to solve multiobjective EED problem and produce multiple Pareto-optimal solutions in one simulation run. In addition, the diversity and well-distribution characteristics of the nondominated solutions obtained by the proposed MOPSO technique have been demonstrated. The simulation results also reveal the superiority of the proposed MOPSO technique in terms of the diversity and quality of the obtained Pareto-optimal solutions.

### Acknowledgment

The author acknowledges the support of King Fahd University of Petroleum & Minerals via funded Project # SAB/2007-01.

### References

- [1] A.A. El-Keib, H. Ma, J.L. Hart, Economic dispatch in view of the clean air act of 1990, *IEEE Trans. Power Syst.* 9 (2) (1994) 972–978.
- [2] J.H. Talaq, F. El-Hawary, M.E. El-Hawary, A summary of environmental/economic dispatch algorithms, *IEEE Trans. Power Syst.* 9 (3) (1994) 1508–1516.
- [3] J.S. Helsin, B.F. Hobbs, A multiobjective production costing model for analyzing emission dispatching and fuel switching, *IEEE Trans. Power Syst.* 4 (3) (1989) 836–842.
- [4] G.P. Granelli, M. Montagna, G.L. Pasini, P. Marannino, Emission constrained dynamic dispatch, *Electric Power Syst. Res.* 24 (1992) 56–64.
- [5] A. Farag, S. Al-Baiyat, T.C. Cheng, Economic load dispatch multiobjective optimization procedures using linear programming techniques, *IEEE Trans. Power Syst.* 10 (2) (1995) 731–738.
- [6] J.S. Dhillon, S.C. Parti, D.P. Kothari, Stochastic economic emission load dispatch, *Electric Power Syst. Res.* 26 (1993) 186–197.
- [7] C.S. Chang, K.P. Wong, B. Fan, Security-constrained multiobjective generation dispatch using bicriterion global optimization, *IEE Proc. Gener. Transm. Distrib.* 142 (4) (1995) 406–414.
- [8] R. Yokoyama, S.H. Bae, T. Morita, H. Sasaki, Multiobjective generation dispatch based on probability security criteria, *IEEE Trans. Power Syst.* 3 (1) (1988) 317–324.
- [9] D. Srinivasan, C.S. Chang, A.C. Liew, Multiobjective generation schedule using fuzzy optimal search technique, *IEE Proc. Gener. Transm. Distrib.* 141 (1994) 231–241.
- [10] C.M. Huang, H.T. Yang, C.L. Huang, Bi-objective power dispatch using fuzzy satisfaction-maximizing decision approach, *IEEE Trans. Power Syst.* 12 (4) (1997) 1715–1721.
- [11] D.B. Das, C. Patvardhan, New multi-objective stochastic search technique for economic load dispatch, *IEE Proc. Gener. Transm. Distrib.* 145 (6) (1998) 747–752.
- [12] M.A. Abido, A novel multiobjective evolutionary algorithm for environmental/economic power dispatch, *Electric Power Syst. Res.* 65 (April (1)) (2003) 71–81.
- [13] M.A. Abido, Environmental/economic power dispatch using multiobjective evolutionary algorithms, *IEEE Trans. Power Syst.* 18 (November (4)) (2003) 1529–1537.
- [14] J. Kennedy, R. Eberhart, *Swarm Intelligence*, Morgan Kaufmann Publishers, 2001.
- [15] M.R. Al-Rashidi, M.E. El-Hawary, Hybrid particle swarm optimization approach for solving the discrete OPF problem considering the valve loading effects, *IEEE Trans. Power Syst.* 22 (November (4)) (2007) 2030–2038.
- [16] Jong-Bae Park, Ki-Song Lee, Joong-Rin Shin, Kwang Y. Lee, A particle swarm optimization for economic dispatch with nonsmooth cost functions, *IEEE Trans. Power Syst.* 20 (1) (2005) 34–42.
- [17] A.I. Selvakumar, K. Thanushkodi, A new particle swarm optimization solution to nonconvex economic dispatch problems, *IEEE Trans. Power Syst.* 22 (1) (2007) 42–51.
- [18] Lingfeng Wang, Chanan Singh, Environmental/economic power dispatch using fuzzified multi-objective particle swarm optimization algorithm, *Electric Power Syst. Res.* 77 (2007) 1654–1664.
- [19] S. Kitamura, K. Mori, S. Shindo, Y. Izui, Y. Ozaki, Multiobjective Energy Management System Using Modified MOPSO, *IEEE Int. Conf. Syst. Man Cybern.* 4 (2005) 3497–3503.
- [20] J. Hazra, A.K. Sinha, Congestion management using multiobjective particle swarm optimization, *IEEE Trans. Power Syst.* 22 (4) (2007) 1726–1734.
- [21] S. Mostaghim, J. Teich, Strategies for finding good local guides in multiobjective particle swarm optimization (MOPSO), in: *Proceedings of 2003 IEEE Swarm Intelligence Symposium*, Indianapolis, IN, USA, April 2003, pp. 26–33.
- [22] J.G. Vlachogiannis, K.Y. Lee, Determining generator contributions to transmission system using parallel vector evaluated particle swarm optimization, *IEEE Trans. Power Syst.* 20 (4) (2005) 1765–1774.
- [23] C.A.C. Coello, A comprehensive survey of evolutionary-based multiobjective optimization techniques, *Knowledge Inform. Syst.* 1 (3) (1999) 269–308.
- [24] E. Zitzler, L. Thiele, An evolutionary algorithm for multiobjective optimization: the strength pareto approach, *Swiss Federal Institute of Technology, TIK-Report*, No. 43, 1998.
- [25] M.A. Abido, Two-Level of nondominated solutions approach to multiobjective particle swarm optimization, in: *Proceedings of the 2007 Genetic and Evolutionary Computation Conference, GECCO'2007*, London, UK, July 7–11, 2007, pp. 726–733.
- [26] J.N. Morse, Reducing the size of nondominated set: pruning by clustering, *Comput. Operat. Res.* 7 (1–2) (1980) 55–66.
- [27] M.A. Abido, Multiobjective evolutionary algorithms for electric power dispatch problem, *IEEE Trans. Evol. Comput.* 10 (June (3)) (2006) 315–329.