FRACTIONAL DERIVATIVE FILTER FOR IMAGE CONTRAST ENHANCEMENT WITH ORDER PREDICTION

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Abstract

Fractional derivative based techniques have been proposed for preprocessing of digital images. Although these techniques address the texture enhancement and other issues to a certain extent, none of them have proposed a method of determining the fractional order adaptively. In this paper, we propose a Grunwald-Letnikov derivative based fractional derivative mask for image contrast enhancement. The proposed mask is multi-directional thus enhancing the image in several directions in one pass. The regularisation based prediction network learns from the training set of images and determines the fractional order based on the statistics of the image at hand. Also the blur reduction is achieved in a controlled fashion as the fractional order is predicted according to the desired blur improvement. Experimental results with the comparative blur metric show the effectiveness of the proposed novel filter on a wide range of images.

1 Introduction

For almost 300 years, fractional calculus was considered as an interesting, but abstract, mathematical concept. Recently, in several muliti-disciplinary application domains, to accurately reflect the non-local, frequency- and history-dependent properties of power law phenomena, fractional calculus modeling tools are being introduced [1]. It has been demonstrated that the operators with non-integer order can describe dynamical behavior of materials and processes over vast time and frequency scales with very concise and computable models [2]. Lundstrom et al [3] report that in neural systems, multiple time scale adaptation is consistent with fractional order differentiation, such that the neuron's firing rate is a fractional derivative of slowly varying stimulus parameters. The rugged surface of a malignant breast cell nucleus is typical of surfaces that cannot be properly understood using the ordinary calculus but may be amenable to studies using fractional calculus [4]. Also since there is evidence of fractal patterns in materials with disordered micro structure under tensile loads and fractal functions cannot be the solution of classical differential equations, fractional calculus has been used to handle fractal processes [5]. It is also hypothesized in the recent years that the biological visual system employs some orders of fractional discrimination to carry out visual perception. Hungenahally [6] observed that perceptual quality is better when fractional discriminant functions were employed instead of merely improving the SNR and the statistical measures. Recently, dynamic models in chaos theory are being generalized by permitting the state dynamics of the model to assume fractional orders[7]. The fact that fractional systems possess memory justifies this generalization. There is an ever increasing list of such practical applications showcasing the power of fractional calculus [8] and from a little more than two decades, it has been the subject of intensely focused research interest [9]. This stimulated us to consider fractional calculus to be applied to the area of image contrast enhancement. Although traditional techniques are available, the fractional derivative approach to the problems often inspires methods rich in insight and extent than in the classical formulation. Fractional calculus has the necessary flexibility to bring in the aimed enrichment and generalization in this field.

In image processing, early application of fractional order differencing was reported by Chandrasekaran et al [10]. They used fractional differentiation to adjust the neural network weights for range image segmentation. Fractional derivative was employed by Mathieu et al [11] for the edge detection purpose. Lu and Xie [12] use fractional calculus based edge operator for iris boundary detection. They demonstrated that the use of non-integer order derivative enhanced the detection selectivity and immunity to noise. Saradhi et al [13] used the fractional derivative approach to propose a fractional derivative-based edge operator. Yifei Pu et al [14] applied the fractional derivative based filter to enhance the texture of a digital image. They proposed structures and parameters for the individual directional masks. In their approach, the output of eight directions is computed and the maximum is taken as the fractional differential grayscale output. Nakib et al [15] applied fractional derivative to the image thresholding. Amelia Sparavigna et al [16] used the fractional differentiation as a tool to reveal faint objects in astronomical images.

Our aim is to produce an image mask which will enhance the image adaptively based on the image statistics. Also it should be efficient in the sense that it should be computationally less expensive than the existing models. In this paper we show the structure and parameters of such a multi-directional mask for the image contrast enhancement. The mask coefficients are based on the Grunwald Letnikov(GL) definition which is one of the fundamental definitions of the fractional order differentiation based on the difference quotients and in a easily amenable series form:

$$D^{\alpha} f(x) = \lim_{h \to 0} h^{-\alpha} \sum_{m=0}^{\frac{x-a}{h}} (-1)^m \frac{\Gamma(\alpha+1)}{m!\Gamma(\alpha-m+1)} f(x-mh) \tag{1}$$

It involves no direct use of the ordinary derivative or integrals unlike some of the other definitions which gives it an advantage over the other forms in terms of implementation.

2 Design of a Fractional Derivative-based Mask

The basic idea is to develop a convolution mask of a specific fractional order necessary to enhance the image contrast to a specified extent quantifiable by a suitable blur metric. Our proposed fractional derivative based filter is based on the Grunwald Letnikov definition of the fractional derivative shown in Equation(1). This series form easily lends itself to be moulded as an image mask as shown in Figure 1.

G2	H1	A2	B1	C2
F1	G1	A1	C1	D1
E2	E1	P	E1	E2
D1	C1	A1	G1	F1
C2	B1	A2	H1	G2

Figure 1: Fractional Order Filter Coefficients

The coefficients of the mask in a particular direction for eg. $a, A1, A2, A3, \ldots$are the first few terms in the GL definition expansion with appropriate step size based on distance between the pixel centers. The center term P is the sum of the center terms in various directions ie. $P = a + b + c + \ldots.$

The realisation of 0.358 order multi-directional fractional derivative filter and the corresponding filter response and its application result on the test image are shown in Figure 2.

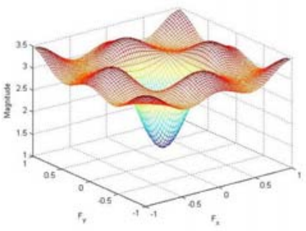
In Figure 3, the image enhancement by the proposed filter for fractional order of 0.38 is shown in comparison with the standard enhancement techniques like contrast stretching and histogram equalisation.

For a specific image, how we arrived at the order of the mask (0.358 & 0.38 in the above mentioned cases)will be explained in the coming sections. The combined multidirectional filter accomplishes the enhancement in all the directions in a single pass. This improves the computational efficiency by a significant amount.

There is a need however to quantify the quality of the image in terms where it could be compared for blur improvement thus enabling the performance comparison.

-0.0236	-0.0801	-0.0267	-0.0801	-0.0236
-0.0801	-0. 0943	-0.1068	-0. 0943	-0.0801
-0.0267	-0.1068	2.9827	-0. 1068	-0.0267
-0.0801	-0.0943	-0. 1068	-0. 0943	-0.0801
-0.0236	-0.0801	-0.0267	-0.0801	-0.0236

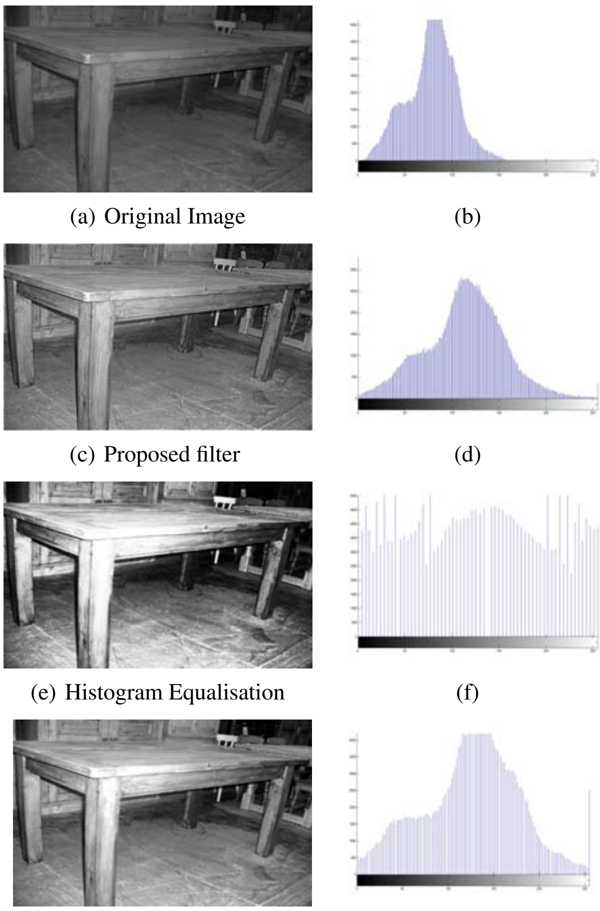
(a) .358 Fractional derivative filter coefficients



(b) Fractional derivative filter frequency response

(c) Original (d) Enhanced

Figure 2: A Sample Application of Fractional Derivative Mask



(a) Original Image (b)

(c) Proposed filter (d)

(e) Histogram Equalisation (f)

(g) Contrast Stretching (h)

Figure 3: Comparison with Standard Techniques

3 The Choice of Blur Metric

Since there is no general theory of image enhancement, when the image is processed for visual interpretation the viewer is the ultimate judge of how well a particular method works. Visual evaluation of image quality is highly subjective thus making the definition of a good image an elusive standard by which to compare an algorithm performance.
Sharpness measures have been traditionally divided into 5 categories namely gradient-based, variance-based, correlation-based, histogram-based, and frequency domain-based methods. Image noise level and artifacts, such as blocking effects, vary with respect to different factors. Conventional sharpness measures are not feasible because they are unable to differentiate variations caused by edges from those induced by image noise and artifacts. Therefore, we need a robust measure which will avoid artificially elevated sharpness values due to image noise and artifacts.
In terms of full-reference image quality measure, PSNR is the most widely used measure. It is appealing because of its simple computation and clear physical meaning. Nevertheless, it is not closely matched to perceived visual quality. Structural similarity index metric (SSIM) based methods [17] are selected for the improved representation of visual perception. This method could be seen as complementary to the traditional PSNR approach but is heavily dependent on the reference image. The Tenengrad metric is the standard metric in the contrast enhancement field. The Tenengrad is based on the Sobel edge detection masks. Jerome Buzzi et al proved in [18] that there is essentially a unique way to quantify blur by a single number and that the variance of the position with respect to the distribution is indeed a unique blur measure. The low frequency phenomena of blur is quantified by the authors as a single number but a quantification scale is not definitely mentioned hence limiting its use in a comparative study. Frederique Crete et al [19] exploit the discrimination between different levels of blur perceptible on the same picture. The advantage of this method is that it does not depend on changing characteristics of image like edge sharpness, or rely on some thresholds but gives an independent and robust estimate of blur on a scale of (0,10) which makes it the blur metric of choice for our application. However, we need to bear in mind that zero blur as defined above is not only counter-intuitive but also difficult to achieve.
Having chosen the above metric, we devised a method of determining the fractional order adaptively based on the image characteristics for a specified reduction of blur value. This is explained in the following section.

4 Regularization

The determination of fractional order for a given image should depend on the image characteristics. It should be able to predict a value tailor-made to the image at hand. Hence, we chose the mean gray scale value, the variance, skewness and the kurtosis of the pixel gray scale values as the image characteristic input since these statistics define the various aspects of the image. We also obtained a variegated set of images which constitute a training set for prediction purpose.
Learning from the training set and attributing a value for a new test sample is cumbersome mainly because the existence of such a value and the sufficiency of the training information are not guaranteed in this case. Also the presence of noise adds uncertainity to the prediction map when this is viewed as a hypersurface reconstruction problem. Hence the prediction of fractional order is an ill posed problem. We make use of Tikhonov's regularization theory to address this problem [20]. Hence the order prediction is done using a regularization network.
The basic idea of regularization is to impose an additional condition like smoothness on this ill-posed problem to stabilize it. To formalize, in our case, we have image x_i and its parameters like its mean grayscale value, their variance and other order statistics. Tikhonov's regularization theory involves two terms - firstly, the standard error term denoted as

$$\mathcal{E}_s(F) = \frac{1}{2}\sum_{i=1}^{N}[d_i - F(x_i)]^2 \tag{2}$$

where $F(x)$ is the approximating function and d_i are the target response and secondly, the regularization term which is denoted as

$$\mathcal{E}_c(F) = \frac{1}{2}\| DF \|^2 \tag{3}$$

where D is a linear differential operator. This operator is embedded with prior information about the form of the solution making its selection problem-specific. It is also referred to as stabiliser because it stabilises the solution to the regularisation problem making it smooth and thereby continuous.
The actual quantity to be minimized in this theory is the Tikhonov functional $\mathcal{E}(F)$ involving the above mentioned terms given as

$$\mathcal{E}(F) = \mathcal{E}_s(F) + \frac{1}{2}\lambda\mathcal{E}_c(F) \tag{4}$$

where λ is a positive real number called the regularisation parameter.
The Euler-Lagrange equation for the Tikhonov functional defines the necessary condition for the Tikhonov functional to have an extremum at $F_\lambda(x)$. This solution $F_\lambda(x)$ turns out to be a linear superposition of N Green's functions as shown.

$$F_\lambda(x) = \frac{1}{\lambda}\sum_{i=1}^{N}[d_i - F(x_i)]G(x, x_i) \tag{5}$$

Further details regarding this network are given in the experimental setup section.

5 Algorithm

Our proposed contrast enhancement scheme is shown in the Figure 4.
The training, validating and testing steps for the proposed regularisation network are as follows-

1. Training

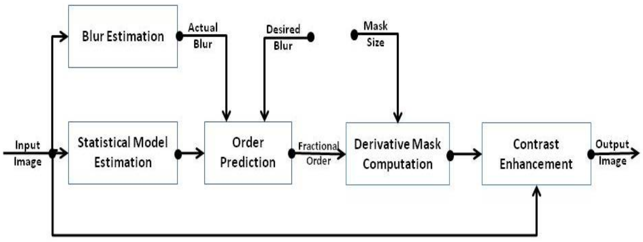


Figure 4: Schematic Diagram

(a) Read the input training images and acquire their first four order statistics and the intitial blur.

(b) Make the fractional order filter mask for various fractional orders and convolve each of the above training images with these and denote the resulting blur value as the desired response d_i for training purposes.

(c) Compute the Green's matrix and from that determine the weight vector mentioned in the Equation 6

2. Validation

   (a) Feed in the training set (seen samples) and compute the deviation of the predicted value from the desired value.

3. Testing and Usage

   (a) For the unseen test samples, compute the first four order statistics and the original blur and feed it to the network alongwith the desired blur.

   (b) For the fractional order predicted using Equation 7, prepare the multi-directional mask, convolve the image and compute the resulting blur value.

6 Experimental Setup, Results and Discussion

For the purpose of training we took 180 images of size 300 x 300 a representative sample of which is shown in Figure 5 including images from [21]. Then we noted the characteristic features of each image namely its first four order statistics (mean, variance, skewness and kurtosis) along with the original blur value. The mask corresponding to various fractional orders was then applied to each image and the change in blur was denoted as the desired blur value.

We used a linear superposition of multivariate Gaussian functions which are rotationally and translationally invariant to be the Green's functions discussed in the regularisation section.

The weights w_i as shown in Figure 6 are defined by

$$w_i = \frac{1}{\lambda}[d_i - F(x_i)] \quad i = 1, 2, \ldots, N \tag{6}$$

Utilising the Green's functions and the weight vector, the appropriate fractional derivative order for a particular test image

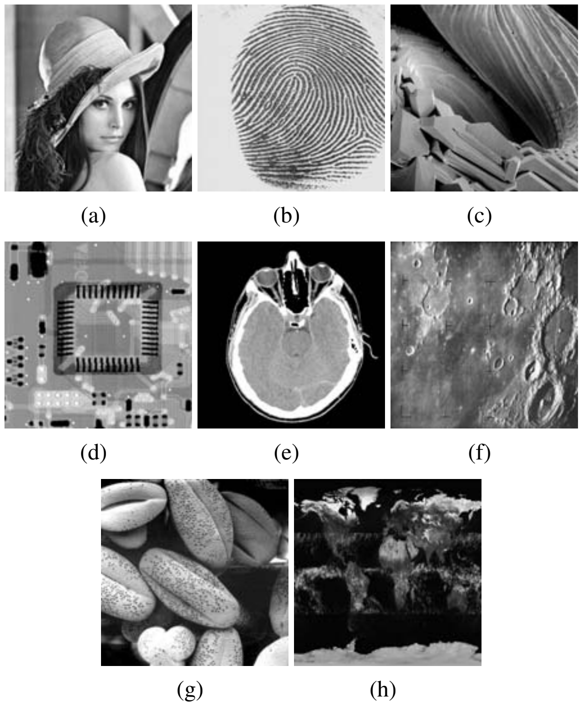


Figure 5: Training Samples

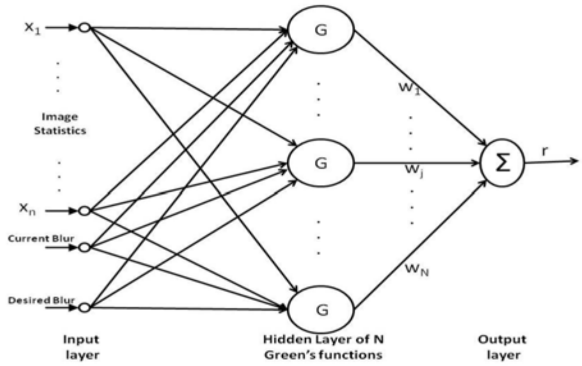


Figure 6: Fractional Order Prediction Network

is computed as shown below.

$$F_\lambda(x) = \sum_{i=1}^{N} w_i exp\left(-\frac{1}{2{\sigma_i}^2} \parallel x - x_i \parallel^2\right) \tag{7}$$

The solution uses all the N training data points to generate the interpolating function F(x). This solution constructs a linear function space that depends on the known data points according to a specified distance measure which in our case is the normalised Euclidean distance measure.

The value of λ in the Equation 6 is an indicator of the sufficiency of data in terms of the training set as examples that determine the solution $F_\lambda(x)$. In other terms, the value of λ shows whether network learns from the samples or predicts based on the smoothness criterion. It can take any value in

$(0, \infty)$. So, if $\lambda \longrightarrow \infty$, it shows that the smoothness criterion imposed on the problem is sufficient to delineate the solution whereas if $\lambda \longrightarrow 0$, it implies that the solution is mainly being determined by the example set.

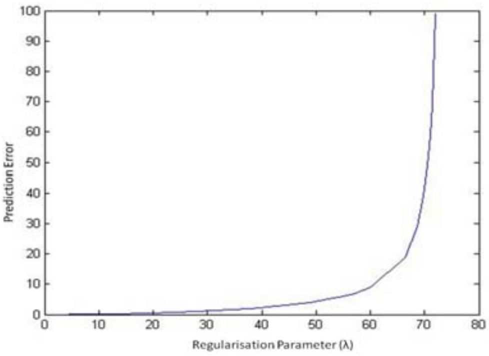


Figure 7: Regularisation Parameter Vs Error in Prediction

In our case, we studied the performance of prediction versus the various values of λ. The graph in Figure 7 show that for the value λ=.01 the performance peaks i.e. in our scenario the network is biased towards learning from the samples.

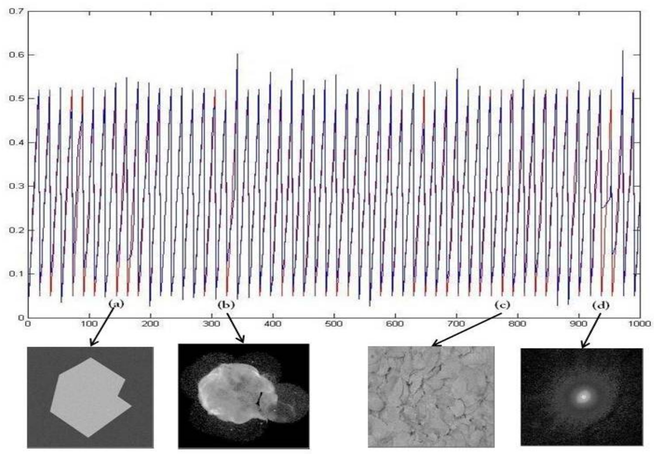
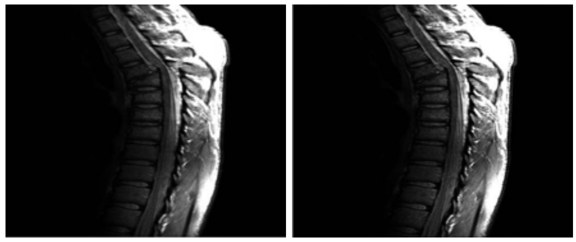


Figure 8: Prediction Performance

We also show the validation results in Figure 8 of fractional order prediction across hundreds of images. Here the red line depicts the desired blur value and the blue line denotes the actual blur value achieved upon prediction. As is clear, the graph shows that the target blur value and the achieved blur value match very closely though as marked on the plot, there are certain regions like (a)-(d) where the match is not exact. Upon analysis, we have found that these are the images in which there was no impressionable change in the blur value for certain fractional orders for the prediction network to learn during the training phase.
In Figure 9, we show some of the images enhanced by using the proposed algorithm. Against each image, the blur value is marked and for the enhanced image the resulting blur against the desired value is indicated. As is clear, the multi-directional fractional derivative mask enhances the image and the blur extent achieved is very close to the blur extent desired.



(a) obl=0.4011 (b) rbl=0.3360, dbl=0.3209

(c) obl=0.3582 (d) rbl=0.2685, dbl=0.2687

(e) obl=0.2641 (f) rbl=0.1873, dbl=0.1849

(g) obl=0.4310 (h) rbl=0.3534, dbl=0.3448

Figure 9: Results: Image (a) is from the Training Set while (b),(c) and (d) are Unseen Test Images. (obl=original blur level,rbl=resultant blur level, dbl=desired blur level)

7 Conclusion

This paper presented a new method for the image enhancement. Using the Grunwald Letnikov fractional derivative, a novel multi-directional mask is proposed and its performance is demonstrated on a variety of real world images. As is evident, for any given arbitrary image for contrast enhancement, the selection of the appropriate fractional order for the mask is an ill-posed problem. In this paper, we have modeled the order prediction for a desired blur reduction as a multi-dimensional surface fitting problem utilising a regularisation network. This network was trained on large classes of images and the contrast enhanced images are improved to the desired extent. The blur values of these enhanced images were determined to be of the desired value as per the comparative blur metric. We demonstrated through examples that our multi-directional mask improves the image contrast in several directions in one pass effectively as evaluated by the blur metric and is unique in the sense that it is possible to enhance to a specified extent.

We consider extending the proposed order prediction based contrast enhancement to the color images in future where the interpretation of contrast in various color spaces as well as the quantification of blur need to be discussed differently. The performance of the current method also is significantly dependent on the efficacy of the chosen blur metric which if perfected in course of future research can be a source of tremendous improvement.

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