

# PID State Feedback Controller of a Quarter Car Active Suspension System

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## ABSTRACT

This paper presents state feedback controller of a quarter-car active suspension system. The controller structure of the active suspension system was decomposed into two loops namely outer loop and inner loop controllers. Outer loop controller is used to calculate the optimum target force to eliminate the effects of road disturbances, while, the inner loop controller is used to keep the actual force close to this desired force. The results of the study show that the inner loop controller is able to track well the target force ranging from sinusoidal to random functions of target force. The performance of outer loop controller also shows significant improvement in terms of body acceleration, body displacement and suspension displacement as compared to the passive suspension system.

**KEYWORDS:** active suspension, hydraulic actuator, forces tracking control, state feedback control.

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## I. INTRODUCTION

Active Suspensions systems have been widely studied over the last 30 years, with hundreds of papers published [1]. Most of the published works focus on the outer-loop controller in computation of the desired control force as a function of vehicle states and the road disturbance [2]. It is commonly assumed that the hydraulic actuator is an ideal force generator and able to carry out the commanded force accurately. Simulations of these outer-loop controllers were frequently done without considering actuator dynamics, or with highly simplified hydraulic actuator dynamics. In real implementation, actuator dynamics can be quite complicated, and the interaction between the actuator and the vehicle suspension cannot be ignored. It is also difficult to produce the actuator force close to the target force without implementing inner-loop or force tracking controller. This is due to the fact that hydraulic actuator exhibits non-linear behavior resulted from servo-valve dynamics, residual structural damping, and the unwanted effects of back-pressure due to the interaction between the hydraulic actuator and vehicle suspension system. A few previous works on the force tracking controller of hydraulic actuator can be found on [2], [3], [4],[5].

This paper focuses on the development of a non-linear hydraulic actuator model including its force tracking controller for an active suspension system. The non-linear hydraulic actuator model consists of servo-valve dynamics and the interaction of piston-cylinder. Force tracking control of the hydraulic actuator model is then performed using Proportional Integral (PI) controller and three constants value must multiplied by variables of quarter car for a variety of the functions of target forces namely step, sinusoidal, saw-tooth, square and random functions. Once the inner loop controller of hydraulic actuator is able to track well the target forces with acceptable error, the hydraulic actuator model and the inner loop controller are then integrated with the outer loop of active suspension control. In this configuration, the inner loop controller must be able to track the optimum target force of hydraulic actuator calculated by the outer loop controller. The actual force of hydraulic actuator is inserted to the vehicle model to reject the effects of road disturbance to the vehicle dynamics performances [5].

This paper is organized as follows: the first section contains introduction, the second section describes the equations of motion of the hydraulic actuator model, the third section presents force tracking control of the hydraulic actuator model, the fourth section elaborates the disturbance rejection control of the active suspension system and the last section presents some conclusions.

## II. MATERIALS AND METHODS

### QUARTER CAR MODEL

The vehicle model considered in this study is a quarter car model. The quarter car model for passive suspension system consists of one-fourth of the body mass, suspension components and one wheel as shown in Figure 1(a). The quarter car model for active suspension system, where the hydraulic actuator is installed in parallel with the spring, is shown in Figure 1(b).

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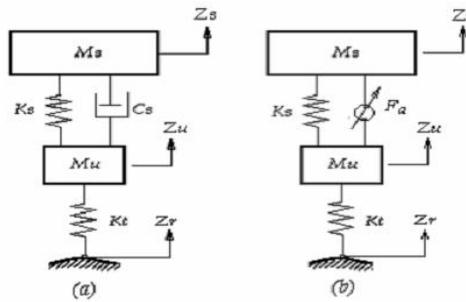


Fig. 1 Passive and active quarter car model

The assumptions of a quarter car modeling are as follows: the tire is modeled as a linear spring without damping, there is no rotational motion in wheel and body, the behavior of spring and damper are linear, the tire is always in contact with the road surface and effect of friction is neglected so that the residual structural damping is not considered into vehicle modeling. The equations of motion for the sprung and unsprung masses of the passive quarter car model are given by

$$\begin{aligned}
 M_s \ddot{Z}_s + K_s (Z_s - Z_u) + C_s (\dot{Z}_s - \dot{Z}_u) &= 0 \\
 M_u \ddot{Z}_u + K_t (Z_u - Z_r) + K_s (Z_u - Z_s) + C_s (\dot{Z}_u - \dot{Z}_s) &= 0
 \end{aligned}
 \tag{1}$$

Whereas, the equations of motion for the sprung and unsprung masses of the semi-active quarter-car model are given by

$$\begin{aligned}
 M_s \ddot{Z}_s + K_s (Z_s - Z_u) + C_s (\dot{Z}_s - \dot{Z}_u) + F_a &= 0 \\
 M_u \ddot{Z}_u + K_t (Z_u - Z_r) + K_s (Z_u - Z_s) + C_s (\dot{Z}_u - \dot{Z}_s) - F_a &= 0
 \end{aligned}
 \tag{2}$$

where  $M_s$  is sprung mass,  $K_s$  is spring stiffness,  $M_u$  is sprung mass,  $C_s$  is damping constant,  $Z_r$  is road profile,  $K_t$  is tire stiffness,  $Z_u$  is unsprung mass displacement,  $Z_s$  is sprung mass displacement and  $F_a$  is actuator force.

The performance criteria of the suspension system to be investigated in this study are body acceleration ( $\ddot{Z}_s$ ), body displacement ( $Z_s$ ), suspension working space ( $Z_s - Z_u$ ) and wheel displacement ( $Z_u$ ). Performance of the suspension system is characterized by the ability of the suspension system in reducing those four performance criteria effectively.

### HYDRAULIC ACTUATOR MODEL

A complete set of a hydraulic actuator consists of five main components namely electro hydraulic powered spool valve, piston-cylinder, hydraulic pump, reservoir and piping system. Power supply is needed to drive the hydraulic pump through AC motor and to control the spool valve position. The spool valve position will control the fluid to come-in or come-out to the piston-cylinder which determines the amount of force produced by the hydraulic actuator. The hydraulic actuators are governed by electro hydraulic servo valve allowing for the generation of forces between the sprung and unsprung masses. The electro hydraulic system consists of an actuator, a primary power spool valve and a secondary bypass valve. As seen in Figure 2, the hydraulic actuator cylinder lies in a follower configuration to a critically centered electro hydraulic power spool valve with matched and symmetric orifices. Positioning of the spool  $u_1$  directs high pressure fluid flow to either one of the cylinder chambers and connects the other chamber to the pump reservoir. This flow creates a pressure difference  $P_L$  across the piston. This pressure difference multiplied by the piston area  $A_p$  is what provides the active force  $F_A$  for the suspension system.

Dynamics for the hydraulic actuator valve are given in equation (3), the change in force is proportional to the position of the spool with respect to center, the relative velocity of the piston, and the leakage through the piston seals. A second input  $u_2$  may be used to bypass the piston component by connecting the piston chambers [6].

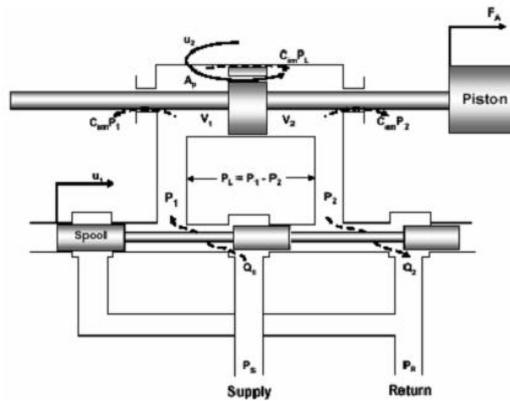


Fig. 2 Physical schematic and variables for the hydraulic actuator [6]

$$\begin{aligned} \dot{F}_a = & A_p \alpha \{ C_{d1} w u_1 \sqrt{\frac{P_s - \text{sgn}(u_1) P_L}{\rho}} \\ & - C_{d2} w u_2 \text{sgn}(P_L) \sqrt{\frac{2 P_L}{\rho}} - C_{im} P_L - A_p (\dot{x}_s - \dot{x}_u) \} \end{aligned} \quad (3)$$

The bypass valve  $u_2$  could be used to reduce the energy consumed by the system. If the spool position  $u_1$  is set to zero, the bypass valve and actuator will behave similar to a variable orifice damper. Spool valve positions  $u_1$  and  $u_2$  are controlled by a current-position feedback loop. The essential dynamics of the spool have been shown to resemble a first order system as the followings

$$\tau \dot{u} + u = kv \quad (4)$$

### CONTROLLER DESIGN

Basically, the controller structure of suspension system utilizes two controller loops namely outer loop, intermediate loop and inner loop controllers as shown in Figure 3. The similar term, which is often used for outer and inner loop controllers, are global and local controllers. The controller structure was used for an active suspension system in [4],[5].

The controller structure adopted in this study is shown in Figure 9. The outer loop controller is used for disturbance rejection control to reduce unwanted vehicle's motions. The inputs of the outer loop controller are vehicle's states namely body velocity and wheel velocity, whereas the output of the outer loop controller is the target force that must be tracked by the hydraulic actuator. On the other hand, the inner loop controller is used for force tracking control of the hydraulic actuator in such a way that the force produced by the hydraulic actuator is as close as possible with the target force produced by the disturbance rejection control.

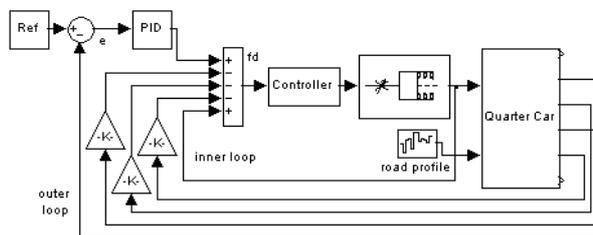


Fig. 3 The controller structure of suspension system

#### A. Inner loop controller

In this configuration, the inner loop controller must be able to track the optimum target force of the hydraulic actuator. The PI controller is implemented by taking force tracking error as the input and delivers control current to drive the spool valve.

The overall equation of inner loop similar in [7] is given by

$$f_d = e.PID - k_1 \dot{Z}_s - k_2 (\dot{Z}_s - \dot{Z}_u) - k_3 (Z_s - Z_u) \tag{5}$$

The validation of the force tracking capability is conducted considering sinusoidal, chirp signal, square wave and saw tooth input forcing functions as the desired force as shown in Figure 4.

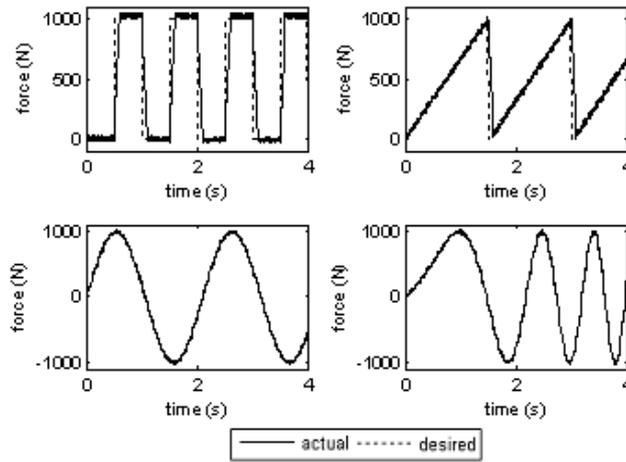


Fig. 4 Force tracking control pneumatic actuator

*B. Outer loop controller*

The disturbance rejection control adopted in this study is a PID controller in which the optimum target force is calculated as the sum of vehicle’s states multiplied by certain feedback gain. The industrial evidence is that for many practical control problems, PID controller is the main control tool being used, because of its simplicity and robust performance [8]. Moreover, the PID control has also been shown to be reliable and stable for operation at low speed and with little disturbances. It is normal that this type of control is located at the outermost loop of a feedback control system, particularly to provide self-nulling capability.

The ideal continuous transfer function of a PID controller is given by [8]

$$PID = K_p \left( e + \frac{1}{T_i} \int_0^t e dt + T_d \frac{de}{dt} \right) \tag{6}$$

where  $T_i = K_p/K_i$ ,  $T_d = K_d/K_p$ , and  $e$  is the error between the reference and the output system,  $T_i$  is the integral time,  $T_d$  is derivative time and  $K_p, K_i, K_d$  are proportional, integral, and derivative gains, respectively.

**III. RESULTS AND DISCUSSIONS**

In this section, the proposed control schemes are implemented and tested within simulation environment of the suspension system and the corresponding results are presented. The suspension system is required to follow a sinusoidal road profile as shown in Figure 5.

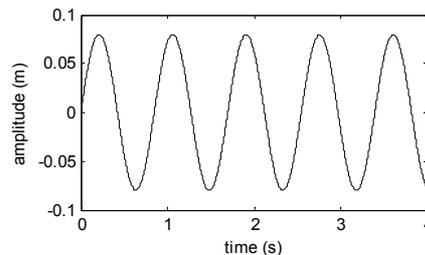


Fig. 5 Road profile configuration

The numerical values of quarter car model parameters are as the followings:

$M_s = 282 \text{ kg}$   
 $M_u = 45 \text{ kg}$   
 $K_s = 17900 \text{ N/m}$   
 $C_s = 1500 \text{ Nsec/m}$   
 $K = 165790 \text{ N/m}$

The simulation was performed for a period of 4 second. The body acceleration and body displacement performances of active system compared with passive system are shown in Figure 6. From the figures, it is clear that the active system is able to significantly reduce both amplitude and the settling time of unwanted body motions in the forms of body acceleration and body displacement as compared with the passive system.

The similar trend was found on the suspension deflection performance as shown in Figure 6, in which the active system shows significant performance in reducing both amplitude and the settling time compared with the passive system. It is also noted that the active system is able to improve the rattle-space dynamics of the suspension system.

In term of the wheel displacement, it can be seen that the magnitude of the wheel displacement for the active system is significantly improvement than the passive system as shown in Figure 6.

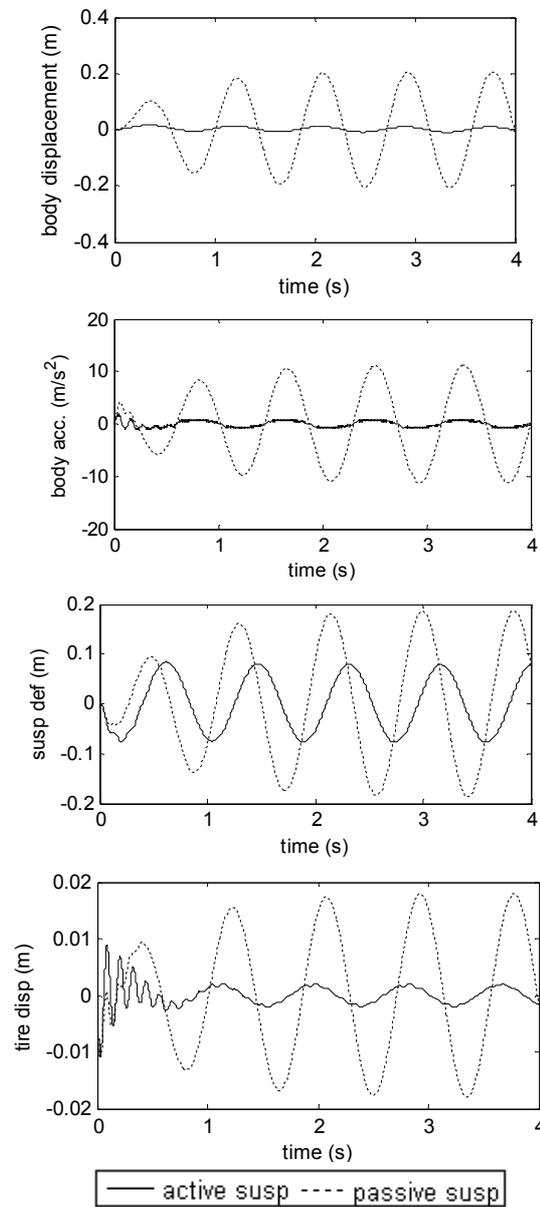


Fig. 6 Passive and active suspension response

#### IV. CONCLUSIONS

The paper presents modeling and force tracking control of hydraulic actuator to be used for an active suspension system. Proportional Integral control was implemented for force tracking control of the hydraulic actuator. The results of the study show that the hydraulic actuator is able to provide the actual force close to the target force with acceptable force tracking error.

A PID state feedback controller was used to reject the effects of road disturbance to the vehicle dynamics performance. From the simulation results, it can be seen that the limited state feedback controller shows significant improvement in reducing both magnitude and settling time of the body acceleration, body displacement and suspension displacement. In term of the wheel displacement, it is noted that even though the magnitude of the wheel displacement for the active system is slightly worse than passive system, the settling time of wheel-hop for the active system is better than passive system.

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