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Least squares based iterative parameter estimation algorithm for multivariable controlled ARMA system modelling with finite measurement data[☆]

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ABSTRACT

Difficulties of identification for multivariable controlled autoregressive moving average (ARMA) systems lie in that there exist unknown noise terms in the information vector, and the iterative identification can be used for the system with unknown terms in the information vector. By means of the hierarchical identification principle, those noise terms in the information vector are replaced with the estimated residuals and a least squares based iterative algorithm is proposed for multivariable controlled ARMA systems. The simulation results indicate that the proposed algorithm is effective.

1. Introduction

System identification is an important approach to model dynamical systems and has been used in many areas such as chemical processes [1], and signal processing [2]. Several methods have been developed for system identification, e.g., the least squares methods [3], gradient based methods [4], the maximum likelihood methods [5] and the step response based method [6,7]. Some useful techniques are used in system identification. For example, the polynomial transformation technique is used to deal with the dual-rate sampled-data systems and the systems with missing observations [8]; the auxiliary model identification idea is used to handle the cases that the information vector contains unknown intermediate variables [9–11]; the hierarchical identification principle is used to reduce the computational cost [12–15]; the multi-innovation identification theory [16–27] and the iterative identification method [11,28–31] make sufficient use of all input–output data and can improve the parameter estimation accuracy.

The least squares based and gradient based iterative methods have been used to solve some matrix equations [32–42]. Also, the iterative methods are very useful for system identification, e.g., Ding et al. proposed a least squares based and a gradient based iterative identification method for OE and OEMA systems [11], and presented a least squares based iterative algorithm for Hammerstein nonlinear ARMAX systems [28]. Liu et al. developed a least squares based iterative identification method for a class of multirate sampled-data systems [31]. Han et al. gave a hierarchical least squares based iterative identification algorithm for a class of multivariable CARMA-like systems [15]. In this paper, we propose a least squares

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based iterative identification method for multivariable controlled ARMA systems. The multivariable model considered in this paper is different from the model in [15].

The rest of this paper is organized as follows. Section 2 derives a least squares based iterative algorithm for the multivariable controlled ARMA systems and gives the identification steps in detail. Section 3 provides a simulation example to show the effectiveness of the proposed algorithm. Finally, concluding remarks are given in Section 4.

2. The derivation of identification algorithm

Consider a multivariable system described by the following controlled ARMA model (multivariable CARMA model for short),

$$\boldsymbol{A}(z)\boldsymbol{y}(t)=\boldsymbol{B}(z)\boldsymbol{u}(t)+\boldsymbol{D}(z)\boldsymbol{v}(t), \tag{1}$$

where $\boldsymbol{u}(t)=[u_1(t),u_2(t),\ldots,u_r(t)]^{\mathrm{T}}\in\mathbb{R}^r$ is the system input vector, $\boldsymbol{y}(t)=[y_1(t),y_2(t),\ldots,y_m(t)]^{\mathrm{T}}\in\mathbb{R}^m$ the system output vector and $\boldsymbol{v}(t)=[v_1(t),v_2(t),\ldots,v_m(t)]^{\mathrm{T}}\in\mathbb{R}^m$ the white noise vector with zero mean, z^{-1} is a unit delay operator: $z^{-1}\boldsymbol{y}(t)=\boldsymbol{y}(t-1)$, $\boldsymbol{A}(z)$, $\boldsymbol{B}(z)$ and $\boldsymbol{D}(z)$ are matrix-coefficient polynomials in z^{-1} with degrees n_a, n_b and n_d, respectively, and

$$\boldsymbol{A}(z)=\boldsymbol{I}+\boldsymbol{A}_1z^{-1}+\boldsymbol{A}_2z^{-2}+\cdots+\boldsymbol{A}_{n_a}z^{-n_a},$$
$$\boldsymbol{B}(z)=\boldsymbol{B}_1z^{-1}+\boldsymbol{B}_2z^{-2}+\cdots+\boldsymbol{B}_{n_b}z^{-n_b},$$
$$\boldsymbol{D}(z)=\boldsymbol{I}+\boldsymbol{D}_1z^{-1}+\boldsymbol{D}_2z^{-2}+\cdots+\boldsymbol{D}_{n_d}z^{-n_d}.$$

$\boldsymbol{A}_i\in\mathbb{R}^{m\times m}$, $\boldsymbol{B}_i\in\mathbb{R}^{m\times r}$ and $\boldsymbol{D}_i\in\mathbb{R}^{m\times m}$ are the matrix coefficients to be estimated. Assume that the orders n_a, n_b and n_d are known and $\boldsymbol{u}(t)=\boldsymbol{0}$, $\boldsymbol{y}(t)=\boldsymbol{0}$ and $\boldsymbol{v}(t)=\boldsymbol{0}$ as $t\leqslant 0$.

The goal of this paper is to present an iterative algorithm to estimate the matrices $\boldsymbol{A}_i$, $\boldsymbol{B}_i$ and $\boldsymbol{D}_i$ from the measured inputs and outputs $\{u(t),y(t): t=1,2,\ldots,L\}$ (L denotes the data length), using the least squares principle.

Let T be the matrix transpose. Define the parameter matrix $\boldsymbol{\theta}$ and the information vector $\boldsymbol{\varphi}(t)$ as

$$\boldsymbol{\theta}^{\mathrm{T}}:=[\boldsymbol{A}_1,\boldsymbol{A}_2,\ldots,\boldsymbol{A}_{n_a},\boldsymbol{B}_1,\boldsymbol{B}_2,\ldots,\boldsymbol{B}_{n_b},\boldsymbol{D}_1,\boldsymbol{D}_2,\ldots,\boldsymbol{D}_{n_d}]\in\mathbb{R}^{m\times n},$$
$$\begin{aligned}\boldsymbol{\varphi}(t):=\;&[-\boldsymbol{y}^{\mathrm{T}}(t-1),-\boldsymbol{y}^{\mathrm{T}}(t-2),\ldots,-\boldsymbol{y}^{\mathrm{T}}(t-n_a),\boldsymbol{u}^{\mathrm{T}}(t-1),\boldsymbol{u}^{\mathrm{T}}(t-2),\ldots,\boldsymbol{u}^{\mathrm{T}}(t-n_b),\\&\boldsymbol{v}^{\mathrm{T}}(t-1),\boldsymbol{v}^{\mathrm{T}}(t-2),\ldots,\boldsymbol{v}^{\mathrm{T}}(t-n_d)]^{\mathrm{T}}\in\mathbb{R}^n,\; n:=mn_a+rn_b+mn_d\end{aligned} \tag{2}$$

then the system model in (1) can be equivalently written as

$$\boldsymbol{y}(t)=\boldsymbol{\theta}^{\mathrm{T}}\boldsymbol{\varphi}(t)+\boldsymbol{v}(t). \tag{3}$$

Eq. (3) is the identification model for the multivariable system in (1).

Consider the data from $t=1$ to $t=L$, and define the stacked output matrix $\boldsymbol{Y}(L)$, the stacked information matrix $\boldsymbol{\Phi}(L)$ and the stacked white noise matrix $\boldsymbol{V}(L)$ as

$$\boldsymbol{Y}(L):=[\boldsymbol{y}(1),\boldsymbol{y}(2),\ldots,\boldsymbol{y}(L)]\in\mathbb{R}^{m\times L},$$
$$\boldsymbol{\Phi}(L):=[\boldsymbol{\varphi}(1),\boldsymbol{\varphi}(2),\ldots,\boldsymbol{\varphi}(L)]\in\mathbb{R}^{n\times L},$$
$$\boldsymbol{V}(L):=[\boldsymbol{v}(1),\boldsymbol{v}(2),\ldots,\boldsymbol{v}(L)]\in\mathbb{R}^{m\times L}.$$

Note that $\boldsymbol{Y}(L)$ and $\boldsymbol{\Phi}(L)$ contain all the measured data $\{u(t),y(t): t=1,2,\ldots,L\}$. From (3), we have

$$\boldsymbol{Y}(L)=\boldsymbol{\theta}^{\mathrm{T}}\boldsymbol{\Phi}(L)+\boldsymbol{V}(L). \tag{4}$$

Define a quadratic criterion function:

$$\boldsymbol{J}(\boldsymbol{\theta}):=\|\boldsymbol{Y}(L)-\boldsymbol{\theta}^{\mathrm{T}}\boldsymbol{\Phi}(L)\|^2,\qquad \|\boldsymbol{X}\|^2:=\mathrm{tr}[\boldsymbol{X}\boldsymbol{X}^{\mathrm{T}}]. \tag{5}$$

Note that $\boldsymbol{V}(L)$ is a white noise matrix with zero mean. For the optimization problem in (5), minimizing $\boldsymbol{J}(\boldsymbol{\theta})$ and letting its partial derivative with respect to $\boldsymbol{\theta}$ be zero give

$$\frac{\partial\boldsymbol{J}(\boldsymbol{\theta})}{\partial\boldsymbol{\theta}}=-2[\boldsymbol{Y}(L)-\boldsymbol{\theta}^{\mathrm{T}}\boldsymbol{\Phi}(L)]\boldsymbol{\Phi}^{\mathrm{T}}(L)=\boldsymbol{0}.$$

Assume that the information vector $\boldsymbol{\varphi}(t)$ is persistently exciting, that is, $[\boldsymbol{\Phi}(L)\boldsymbol{\Phi}^{\mathrm{T}}(L)]$ is an invertible matrix, then from above equation, we can obtain the least squares estimate (LSE) of $\boldsymbol{\theta}$:

$$\begin{aligned}\hat{\boldsymbol{\theta}}&=[\boldsymbol{\Phi}(L)\boldsymbol{\Phi}^{\mathrm{T}}(L)]^{-1}\boldsymbol{\Phi}(L)\boldsymbol{Y}^{\mathrm{T}}(L)\\&=\left[\sum_{t=1}^{L}\boldsymbol{\varphi}(t)\boldsymbol{\varphi}^{\mathrm{T}}(t)\right]^{-1}\sum_{t=1}^{L}\boldsymbol{\varphi}(t)\boldsymbol{y}^{\mathrm{T}}(t).\end{aligned} \tag{6}$$

However, from (2), we can see that the information vector $\boldsymbol{\varphi}(t)$ $(t = 1, 2, \ldots, L)$ contain the unmeasurable noise terms $\boldsymbol{v}(t-i)$ $(i = 1, 2, \ldots, n_d)$, thus Eq. (6) cannot give the estimate $\hat{\boldsymbol{\theta}}$ directly. The commonly used method is to replace the unmeasurable noise terms with their estimated residuals. In this paper, we propose an iterative identification method using the hierarchical identification principle. Let $k = 1, 2, 3, \ldots$ be an iteration variable, and $\hat{\boldsymbol{\theta}}_k$ be the estimate of $\boldsymbol{\theta}$ at iteration k, $\hat{\boldsymbol{\varphi}}_k(t)$ denote the information vector obtained by replacing the inner unknown $\boldsymbol{v}(t-i)$ in $\boldsymbol{\varphi}(t)$ with the estimate $\hat{\boldsymbol{v}}_{k-1}(t-i)$ at iteration $k-1$, and $\hat{\boldsymbol{\Phi}}_k(L)$ denote the stacked information matrix obtained by replacing $\boldsymbol{\varphi}(t)$ in $\boldsymbol{\Phi}(L)$ with $\hat{\boldsymbol{\varphi}}_k(t)$, i.e.,

$$\begin{aligned}\hat{\boldsymbol{\varphi}}_k(t) &:= [-\boldsymbol{y}^{\mathrm{T}}(t-1), -\boldsymbol{y}^{\mathrm{T}}(t-2), \ldots, -\boldsymbol{y}^{\mathrm{T}}(t-n_a), \boldsymbol{u}^{\mathrm{T}}(t-1), \boldsymbol{u}^{\mathrm{T}}(t-2), \ldots, \boldsymbol{u}^{\mathrm{T}}(t-n_b),\\ &\quad\ \hat{\boldsymbol{v}}^{\mathrm{T}}_{k-1}(t-1), \hat{\boldsymbol{v}}^{\mathrm{T}}_{k-1}(t-2), \ldots, \hat{\boldsymbol{v}}^{\mathrm{T}}_{k-1}(t-n_d)]^{\mathrm{T}} \in \mathbb{R}^n,\\ \hat{\boldsymbol{\Phi}}_k(L) &:= [\hat{\boldsymbol{\varphi}}_k(1), \hat{\boldsymbol{\varphi}}_k(2), \ldots, \hat{\boldsymbol{\varphi}}_k(L)] \in \mathbb{R}^{n\times L}.\end{aligned} \tag{7}$$

From (3), we have

$$\boldsymbol{v}(t) = \boldsymbol{y}(t) - \boldsymbol{\theta}^{\mathrm{T}}\boldsymbol{\varphi}(t).$$

If $\boldsymbol{\varphi}(t)$ and $\boldsymbol{\theta}$ are replaced with their estimates $\hat{\boldsymbol{\varphi}}_k(t)$ and $\hat{\boldsymbol{\theta}}_k$, then the estimate of $\boldsymbol{v}(t)$ at iteration k can be computed by

$$\hat{\boldsymbol{v}}_k(t) = \boldsymbol{y}(t) - \hat{\boldsymbol{\theta}}^{\mathrm{T}}_k\hat{\boldsymbol{\varphi}}_k(t). \tag{8}$$

Replacing $\boldsymbol{\Phi}(L)$ in (6) with $\hat{\boldsymbol{\Phi}}_k(L)$ gives the least squares based iterative parameter estimation algorithm for the multivariable CARMA systems (CARMA-LSI):

$$\hat{\boldsymbol{\theta}}_k = [\hat{\boldsymbol{\Phi}}_k(L)\hat{\boldsymbol{\Phi}}^{\mathrm{T}}_k(L)]^{-1}\hat{\boldsymbol{\Phi}}_k(L)\boldsymbol{Y}^{\mathrm{T}}(L), \quad k = 1, 2, 3, \ldots \tag{9}$$

$$\hat{\boldsymbol{\Phi}}_k(L) = [\hat{\boldsymbol{\varphi}}_k(1), \hat{\boldsymbol{\varphi}}_k(2), \ldots, \hat{\boldsymbol{\varphi}}_k(L)], \tag{10}$$

$$\boldsymbol{Y}(L) = [\boldsymbol{y}(1), \boldsymbol{y}(2), \ldots, \boldsymbol{y}(L)], \tag{11}$$

$$\begin{aligned}\hat{\boldsymbol{\varphi}}_k(t) &= [-\boldsymbol{y}^{\mathrm{T}}(t-1), -\boldsymbol{y}^{\mathrm{T}}(t-2), \ldots, -\boldsymbol{y}^{\mathrm{T}}(t-n_a), \boldsymbol{u}^{\mathrm{T}}(t-1), \boldsymbol{u}^{\mathrm{T}}(t-2), \ldots, \boldsymbol{u}^{\mathrm{T}}(t-n_b),\\ &\quad\ \hat{\boldsymbol{v}}^{\mathrm{T}}_{k-1}(t-1), \hat{\boldsymbol{v}}^{\mathrm{T}}_{k-1}(t-2), \ldots, \hat{\boldsymbol{v}}^{\mathrm{T}}_{k-1}(t-n_d)]^{\mathrm{T}},\end{aligned} \tag{12}$$

$$\hat{\boldsymbol{v}}_k(t) = \boldsymbol{y}(t) - \hat{\boldsymbol{\theta}}^{\mathrm{T}}_k\hat{\boldsymbol{\varphi}}_k(t), \quad t = 1, 2, \ldots, L. \tag{13}$$

In this algorithm, the initial value $\hat{\boldsymbol{v}}_0(t)$ is often chosen as a random vector. From (9)–(13), we can see that the CARMA-LSI algorithm performs a hierarchical interactive process: when computing the parameter estimates $\hat{\boldsymbol{\theta}}_k$, the unknown noise terms $\boldsymbol{v}(t-i)$, $i = 1, 2, \ldots, n_d$, in the information vector are replaced with their corresponding estimates $\hat{\boldsymbol{v}}_{k-1}(t-i)$ at the $k-1$th iteration, while the noise estimates $\hat{\boldsymbol{v}}_k(t)$ at iteration k are computed from the parameter estimates $\hat{\boldsymbol{\theta}}_k$.

The identification steps of the CARMA-LSI algorithm to compute $\hat{\boldsymbol{\theta}}_k(t)$ are listed as follows

1. Collect the input–output data $\{u(t), y(t) : t = 1, 2, \ldots, L\}(L \gg n)$ and form the stacked output matrix $\boldsymbol{Y}(L)$ by (11).
2. Let $k = 1$, set $\hat{\boldsymbol{v}}_0(t)$ a random vector.
3. Form $\hat{\boldsymbol{\varphi}}_k(t)$ by (12), and then form $\hat{\boldsymbol{\Phi}}_k(L)$ by (10).
4. Update the estimate $\hat{\boldsymbol{\theta}}_k$ by (9).
5. Compute $\hat{\boldsymbol{v}}_k(t)$ by (13).
6. Compare $\hat{\boldsymbol{\theta}}_k$ with $\hat{\boldsymbol{\theta}}_{k-1}$, if they are sufficiently close, or for some pre-set small ε, if

   $$\|\hat{\boldsymbol{\theta}}_k - \hat{\boldsymbol{\theta}}_{k-1}\| \leqslant \varepsilon$$

   then terminate this procedure and obtain the iterative time k and estimate $\hat{\boldsymbol{\theta}}_k$; otherwise, increase k by 1 and go to step 3.

   The flowchart of computing the parameter estimate $\hat{\boldsymbol{\theta}}_k$ is shown in Fig. 1.

3. Example

In this section, an example is given to show that the proposed iterative algorithm is effective. Consider the following 2-input and 2-output system:

$$\begin{bmatrix} y_1(t)\\ y_2(t)\end{bmatrix} + \begin{bmatrix} 0.60 & 0.50\\ -0.80 & 1.00\end{bmatrix}\begin{bmatrix} y_1(t-1)\\ y_2(t-1)\end{bmatrix} = \begin{bmatrix} 1.50 & -0.40\\ -0.50 & 1.10\end{bmatrix}\begin{bmatrix} u_1(t-1)\\ u_2(t-1)\end{bmatrix} + \begin{bmatrix} v_1(t)\\ v_2(t)\end{bmatrix} + \begin{bmatrix} 0.20 & -0.10\\ -0.10 & 0.60\end{bmatrix}\begin{bmatrix} v_1(t-1)\\ v_2(t-1)\end{bmatrix}.$$

Here, $\{u_1(t)\}$ and $\{u_2(t)\}$ are taken as persistent excitation signal sequences with zero mean and unit variance, $\{v_1(t)\}$ and $\{v_2(t)\}$ as white noise sequences with zero mean and variances $\sigma_1^2 = \sigma_2^2 = 0.50^2$. Applying the CARMA-LSI algorithm

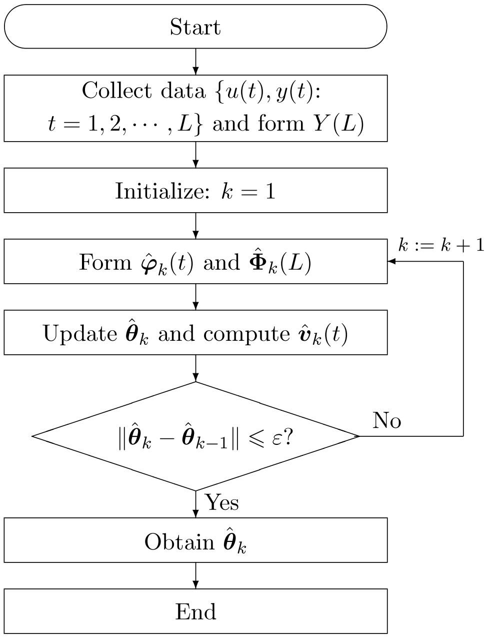

Fig. 1. The flowchart of computing the CARMA-LSI parameter estimate $\hat{\boldsymbol{\theta}}_k$.

Table 1
The CARMA-LSI parameter estimates and errors ($L = 1000$).

k	1	2	3	4	5	6
$a_{11} = 0.60000$	0.59989	0.59991	0.59991	0.59991	0.59991	0.59992
$a_{12} = 0.50000$	0.50009	0.50007	0.50007	0.50007	0.50007	0.50007
$b_{11} = 1.50000$	1.53201	1.52962	1.53039	1.53052	1.53053	1.53053
$b_{12} = -0.40000$	−0.40837	−0.40712	−0.40649	−0.40593	−0.40584	−0.40583
$d_{11} = 0.20000$	0.00440	0.21978	0.23909	0.24087	0.24088	0.24086
$d_{12} = -0.10000$	0.01175	−0.07751	−0.11764	−0.12421	−0.12427	−0.12421
$a_{21} = -0.80000$	−0.79959	−0.79963	−0.79963	−0.79962	−0.79962	−0.79964
$a_{22} = 1.00000$	0.99959	0.99964	0.99963	0.99963	0.99964	0.99963
$b_{21} = -0.50000$	−0.48619	−0.48893	−0.48960	−0.48993	−0.48998	−0.48998
$b_{22} = 1.10000$	1.11032	1.09918	1.09497	1.09308	1.09277	1.09276
$d_{21} = -0.10000$	−0.01132	−0.07535	−0.11506	−0.12093	−0.12093	−0.12092
$d_{22} = 0.60000$	0.01841	0.43624	0.57962	0.60221	0.60238	0.60236
δ (%)	24.67664	6.70177	2.34000	2.41533	2.41777	2.41642

Table 2
The CARMA-LSI parameter estimates and errors ($L = 2000$).

k	1	2	3	4	5	6
$a_{11} = 0.60000$	0.59990	0.59993	0.59993	0.59992	0.59993	0.59993
$a_{12} = 0.50000$	0.50033	0.50031	0.50031	0.50032	0.50031	0.50031
$b_{11} = 1.50000$	1.52074	1.51557	1.51486	1.51460	1.51456	1.51455
$b_{12} = -0.40000$	−0.39568	−0.39719	−0.39705	−0.39683	−0.39679	−0.39678
$d_{11} = 0.20000$	0.01671	0.19748	0.21280	0.21364	0.21360	0.21360
$d_{12} = -0.10000$	−0.01715	−0.07863	−0.11148	−0.11515	−0.11521	−0.11521
$a_{21} = -0.80000$	−0.79991	−0.79994	−0.79995	−0.79993	−0.79993	−0.79994
$a_{22} = 1.00000$	0.99994	1.00000	0.99999	0.99999	1.00000	1.00000
$b_{21} = -0.50000$	−0.49558	−0.48877	−0.48603	−0.48500	−0.48483	−0.48481
$b_{22} = 1.10000$	1.09733	1.09810	1.09686	1.09611	1.09596	1.09594
$d_{21} = -0.10000$	−0.02135	−0.08146	−0.10545	−0.11015	−0.11043	−0.11045
$d_{22} = 0.60000$	0.02569	0.40962	0.52741	0.54755	0.54946	0.54963
δ (%)	24.00801	7.56540	3.03650	2.38901	2.32915	2.32408

in (9)–(13) to estimate the parameter matrix $\boldsymbol{\theta}$ of this system. The parameter estimates and their estimation errors with the data length $L = 1000$, $L = 2000$ and $L = 3000$ are shown in Tables 1–3, where the estimation error is defined as $\delta := \|\hat{\boldsymbol{\theta}}_k - \boldsymbol{\theta}\| / \|\boldsymbol{\theta}\|$.

Table 3
The CARMA-LSI parameter estimates and errors ($L = 3000$).

k	1	2	3	4	5	6
$a_{11} = 0.60000$	0.60005	0.60005	0.60005	0.60005	0.60005	0.60005
$a_{12} = 0.50000$	0.50000	0.50000	0.50000	0.50000	0.50000	0.50000
$b_{11} = 1.50000$	1.50744	1.50452	1.50436	1.50425	1.50423	1.50423
$b_{12} = -0.40000$	−0.41562	−0.41347	−0.41354	−0.41342	−0.41340	−0.41340
$d_{11} = 0.20000$	0.01038	0.16975	0.18386	0.18587	0.18611	0.18613
$d_{12} = -0.10000$	0.00825	−0.05506	−0.08387	−0.09068	−0.09126	−0.09127
$a_{21} = -0.80000$	−0.79995	−0.79996	−0.79997	−0.79996	−0.79996	−0.79997
$a_{22} = 1.00000$	0.99992	0.99994	0.99994	0.99994	0.99994	0.99994
$b_{21} = -0.50000$	−0.51447	−0.50962	−0.50968	−0.50934	−0.50928	−0.50928
$b_{22} = 1.10000$	1.09646	1.09933	1.09930	1.09869	1.09858	1.09858
$d_{21} = -0.10000$	−0.01091	−0.05595	−0.08684	−0.09168	−0.09193	−0.09194
$d_{22} = 0.60000$	0.00119	0.42970	0.56287	0.57790	0.57801	0.57802
δ (%)	25.18160	7.22840	1.90336	1.31543	1.29898	1.29799

From the simulation results in Tables 1–3, we can draw the following conclusions:

1. The estimation errors δ are becoming smaller (in general) as the iterations k increases. Thus the proposed algorithm for multivariable CARMA systems is effective.
2. A longer data length L leads to a smaller estimation error under the same noise level.
3. The CARMA-LSI algorithm converges very fast and needs only a few iterations to converge to their true values.

4. Conclusions

This paper presents a least squares based iterative parameter estimation algorithm for multivariable controlled ARMA systems. The basic idea is to use the iterative technique and to replace the unknown terms in the information vector with their iterative estimates. Since the proposed algorithm makes full use of the measured input–output data, it can provide more accurate parameter estimates than existing recursive algorithms. The proposed algorithm can be extended to identify time-varying systems [43], nonlinear systems [44–48], dual-rate/multirate systems [49–58], as well as to design filters [59–62] and estimate states [63].

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