Nonlinear Backstepping Control of Permanent Magnet Synchronous Motor (PMSM)

M.S. Merzoug [*] **, H. Benalla** [**]

[*] *Department of Department of Electrical Engineering, University of Mentouri Constantine, B.P. 325 , avenue Ain El Bey, Constantine 25017, Algérie*
tel/fax: (213)31.81.87.11
e-mail: merzougmohamedsalah@yahoo.fr

[**] *Department of Department of Electrical Engineering, University of Mentouri Constantine, B.P. 325 , avenue Ain El Bey, Constantine 25017, Algérie*
tel/fax: (213)31.81.87.11
e-mail: Hbenalla@yahoo.dz

Submitted: 08/01/2010
Accepted: 11/01/2010
Appeared: 16/01/2010
©HyperSciences.Publisher

Abstract—This paper presents a novel speed control technique for an permanent magnets synchronous (MSAP) drive based on newly Nonlinear backstepping technique. The most appealing point of it is to use the virtual control variable to make the high-order system simple, and thus the final control outputs can be derived step by step through appropriate Lyapunov functions Backstepping control approach is adapted to derive the control scheme, which is robust to parameter uncertainties and external load disturbance. Simulation results clearly show that the proposed controller can track the speed reference signal successfully under parameter uncertainties and load torque disturbance rejection.

Keywords: PMSM, Backstepping, Lyapunov, Stability, Nonlinear

1. NOMENCLATURE

V_d, V_q	Direct-and quadrature-axis stator voltages
I_d, I_q	Direct-and quadrature-axis stator voltages
L_d, L_q	Direct -and quadrature-axis inductance
P	Number of poles
R_S	Stator resistance
φ_f	rotor magnet flux linkage
φ_d, φ_q	Flux in (d,q) reference frame
ω_r	rotor speed in electrical
Ω	Mechanical rotor speed
θ	Electrical rotor position
T_e	Electromagnetic torque
T_f	Friction torque
T_r	Load torque
J	Inertia
F	Damping coefficient

2. INTRODUCTION

Permanent magnet (PM) synchronous motors have attracted increasing interest in recent years for industrial drive application. The high efficiency, high steady state torque density and simple controller of the PM motor drives compared with the induction motor drives make them a good alternative in certain applications. Moreover, the availability of low-cost power electronic devices and the improvement of PM characteristics enable the use of PM motors even in some more demanding applications [4].

Advantages of PMSM include low inertia, high efficiency, high power density and reliability. Because of these advantages, PMSM are indeed excellent for use in high-performance servo drives where a fast and accurate torque response is required.

Usually, high-performance motor drives require fast and ac- curate response, quick recovery from any disturbances and insensitivity to parameter variations. The dynamic behavior of an ac motor can be significantly improved using vector control theory where motor variables are transformed into an orthogonal set of d–q axes such that speed and torque can be controlled separately. This gives the IPMSM machine the highly desirable dynamic performance capabilities of the separately excited dc machine, while retaining the general advantages of the ac over dc motors. Originally, vector control was applied to the induction motor and a vast amount of research work has been devoted to this area. The vector control method is relevant to the IPMSM drive as the control is completely carried out through the stator, as the rotor excitation control is not possible. [6]

The two major classes of controllers which are capable of dealing with nonlinear uncertain systems are adaptive and robust controllers. Backstepping control is an approach to nonlinear control design which has attracted a great deal of research interest in recent years. It is mainly applicable to systems having a cascaded or triangular structure.

The central idea of the approach is to recursively design controllers for motor torque constant uncertainty subsystems in the structure and ‘‘step back’’ the feedback signals towards the control input. This differs from the conventional feedback linearization in that it can avoid cancellation of useful nonlinearities in pursuing the objectives of stabilization and tracking. In addition, by utilizing the control Lyapunov function, it also has the flexibility in introducing appropriate dynamics to make the system behave in a desired manner. The presentations of backstepping control in the literature are mostly in pure mathematical settings. [2]

The Backstepping control is a systematic and recursive design methodology for nonlinear feedback control. Appling those design methods, control objectives such as position, velocity can be achieved. [1]

A nonlinear backstepping control design scheme is developed for the speed tracking control of PMSM that has exact model knowledge. The asymptotic stability of the resulting closed-loop system is guaranteed according to Lyapunov stability theorem.

3. MATHEMATICAL MODEL OF THE PMSM

The electrical and mechanical equations of the PMSM in the rotor reference (d-q) frame are as follows [5]:

$$\begin{cases} V_d = R_s I_d + \frac{d}{dt}\varphi_d - \omega_r \varphi_q \\ V_q = R_s I_q + \frac{d}{dt}\varphi_q + \omega_r \varphi_d \end{cases} \tag{1}$$

$$\varphi_d = L_d I_d + \varphi_f \tag{2}$$

$$\varphi_q = L_q I_q \tag{3}$$

And the electromagnetic torque T_e is given by:

$$T_e = \frac{3}{2} P[(L_d - L_q) I_d I_q + I_q \varphi_f)] \tag{4}$$

The equation for the motor dynamics, on the other hand, is

$$T_e - T_r - T_f = J\frac{d\Omega}{dt} \tag{5}$$

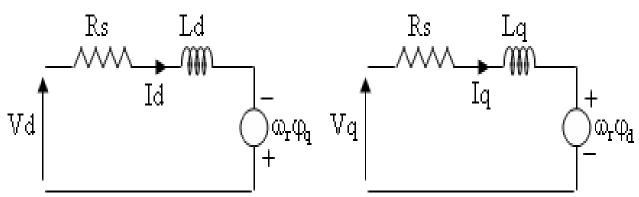

Fig. 1. PMSM equivalent circuit from dynamic equations

From (1), it is obvious that the dynamic model of PMSM is highly nonlinear because of the coupling between the speed and the electrical currents. According to the vector control principle, the direct axis current i_d is always forced to be zero in order to orient all the linkage flux in the d-axis and achieve maximum torque per ampere. With the state assignments, the dynamic $x_1 = I_d$, $x_2 = \omega_r$ and $x_3 = I_q$.

Model of PMSM can be rewritten as follows:

$$\begin{cases} \dot{x}_1 = -\frac{R_s}{L_d} x_1 + \frac{L_q}{L_d} P x_2 x_3 + \frac{1}{L_d} V_d \\ \dot{x}_2 = \frac{P\varphi_f}{J} x_3 - \frac{f}{J} x_2 - \frac{1}{J} Tr \\ \dot{x}_3 = -\frac{R_s}{L_q} x_3 - \frac{L_d}{L_q} P x_2 x_1 - \frac{P}{L_q}\varphi_f x_2 + \frac{1}{L_q} V_q \end{cases} \tag{6}$$

4. NONLINEAR BACKSTEPPING DESIGN

The schematic diagram of the speed control system under study is shown in Fig. 2. The parameters of the synchronous machine are given in the Appendix. In this section, we employ the nonlinear backstepping schemes to design the controllers for PMSM systems with angular velocity measurement.

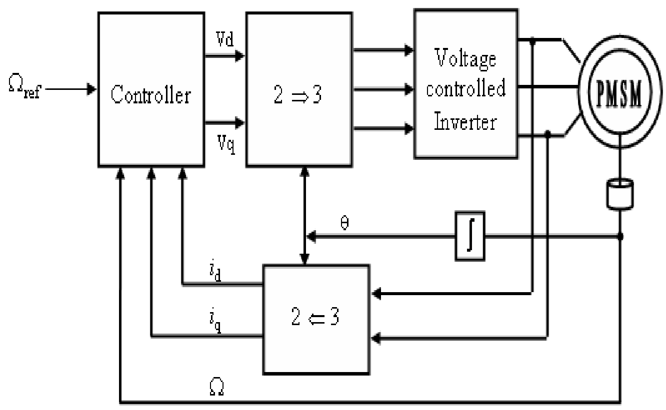

Fig. 2. System configuration of Backstepping control

With the choice of appropriate regulated variables, the backstepping design procedure consists of the following three steps : [7]

Step 1: First of all, since the direct axis current i_d must be forced to be zero, the first regulated variable is introduced by

$$z_1 = x_1 \tag{7}$$

The derivative of (7) is computed as

$$\begin{aligned} \dot{z}_1 &= \dot{x}_1 \\ &= -\frac{R_s}{L_d} x_1 + \frac{L_q}{L_d} P x_2 x_3 + \frac{1}{L_d} V_d \end{aligned} \tag{8}$$

The first Lyapunov candidate V_1 is chosen as:

$$V_1 = \frac{1}{2} z_1^2 \tag{9}$$

So the derivative of (9) is computed as:

$$\dot{V}_1 = z_1\dot{z}_1 = z_1\left(-\frac{R_s}{L_d}z_1 + \frac{L_q}{L_d}Px_2x_3 + \frac{1}{L_d}V_d\right) \tag{10}$$

At this point, the direct axis voltage control input V_d can be selected by

$$V_d = -L_d\left(c_1z_1 + \frac{L_q}{L_d}Px_2x_3\right) \tag{11}$$

Where c_1 is a positive design constant, so (8) becomes:

$$\dot{z}_1 = -(c_1 + \frac{R}{L_d})z_1 \tag{12}$$

Therefore, (10) can be rewritten as:

$$\dot{V}_1 = -(c_1 + \frac{R}{L_d})z_1^2 \tag{13}$$

Step 2: The purpose of this control design is to achieve the reference speed tracking, so the second regulated variable is chosen as

$$z_2 = x_2 - \Omega_{ref} \tag{14}$$

Where Ω_{ref} is the speed reference, hence the derivative of (14) is calculated as:

$$\dot{z}_2 = \frac{P\varphi_f}{J}x_3 - \frac{f}{J}x_2 - \frac{1}{J}Tr - \dot{\Omega}_{ref} \tag{15}$$

By defining the error variable $z_3 = x_3 - \alpha$ where α is the stabilizing function chosen as follows:

$$\alpha = \frac{J}{P\varphi_f}\left(\frac{f}{J}\Omega_{ref} + \frac{1}{J}Tr + \dot{\Omega}_{ref}\right) \tag{16}$$

(15) can be rewritten as

$$\dot{z}_2 = -\frac{f}{J}z_2 + \frac{P\varphi_f}{J}z_3 \tag{17}$$

With the choice of the second Lyapunov candidate

$$V_2 = \frac{1}{2}c_2z_2^2 \tag{18}$$

where c_2 is a positive design constant, the derivative of (18) is computed as :

$$\dot{V}_2 = c_2z_2\dot{z}_2 = -\frac{f}{J}c_2z_2^2 + \frac{P\varphi_f}{J}c_2z_2z_3 \tag{19}$$

Step 3: The derivative of the given error variable z_3 is computed as

$$\dot{z}_3 = -\frac{R_s}{L_q}x_3 - \frac{L_d}{L_q}Px_2x_1 - \frac{P}{L_q}\varphi_f x_2 + \frac{1}{L_q}V_q - \frac{J}{P\varphi_f}\left(\frac{f}{J}\dot{\Omega}_{ref} + \ddot{\Omega}_{ref}\right) \tag{20}$$

With the selection of the complete Lyapunov function

$$V = V_1 + V_2 + \frac{1}{2}z_3^2 \tag{21}$$

From (13), (19) and (20) and the derivative of (21) is computed as follows:

$$\begin{aligned} V &= \dot{V}_1 + \dot{V}_2 + z_3\dot{z}_3 \\ &= -(c_1 + \frac{R}{L_d})z_1^2 - \frac{f}{J}c_2z_2^2 \\ &+ z_3[\frac{P\varphi_f}{J}c_2z_2 - \frac{R_s}{L_q}x_3 - \frac{L_d}{L_q}Px_2x_1 - \frac{P}{L_q}\varphi_f x_2 \\ &+ \frac{1}{L_q}V_q - \frac{J}{P\varphi_f}(\frac{f}{J}\dot{\Omega}_{ref} + \ddot{\Omega}_{ref})] \end{aligned} \tag{22}$$

At last, in order to make the derivative of the complete Lyapunov function (21) be negative definite, the *q*-axis voltage control input is chosen as follows:

$$\begin{aligned} V_q = L_q[&-\frac{P\varphi_f}{J}c_2z_2 + \frac{R_s}{L_q}x_3 + \frac{L_d}{L_q}Px_2x_1 + \frac{P}{L_q}\varphi_f x_2 \\ &+ \frac{J}{P\varphi_f}(\frac{f}{J}\dot{\Omega}_{ref} + \ddot{\Omega}_{ref}) - c_3z_3] \end{aligned} \tag{23}$$

Therefore, substituting (23) into (22), we are able to obtain

$$\dot{V} = -(c_1 + \frac{R}{L_d})z_1^2 - \frac{f}{J}c_2z_2^2 - c_3z_3^2 \tag{24}$$

Clearly, in (24) is negative definite, so it implies that the resulting closed-loop system is asymptotically stable and, hence, all the error variables z_1, z_2 and z_3 will converge to zero asymptotically.

5. VOLTAGE SOURCE PWM INVERTER

A typical voltage-source PWM converter performs the ac to

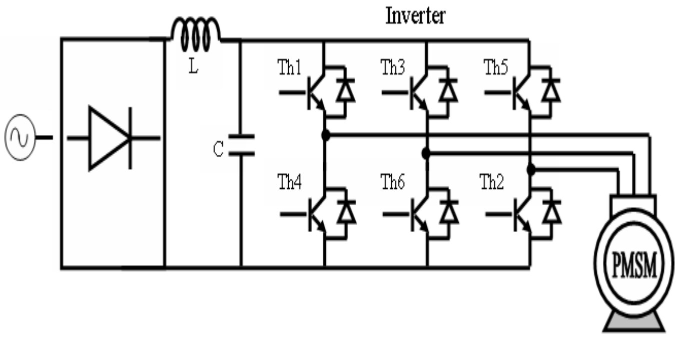

Fig. 3. Basic three-phase voltage-source converter circuit

5.1. Sinusoidal pulse width modulation

Three-phase reference voltages of variable amplitude and frequency are compared in three separate comparators with a common triangular carrier wave of fixed amplitude and frequency Fig. 4. Each comparator output forms the switching-state of the corresponding inverter leg. [3]

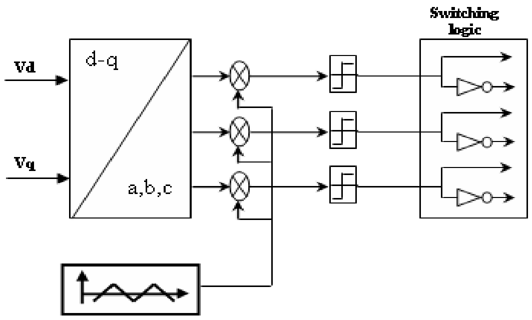

Fig. 4. Sinusoidal PWM, current control and switching logic

6. SIMULATION RESULTS

Extensive simulations have been performed using Matlab/ Simulink Software to examine control algorithm of the nonlinear backstepping applied for PMSM.

In order to validate the control strategies as discussed, digital simulation studies were made the system described in Fig. 2. The speed and currents loops of the drive were also designed and simulated respectively with backstepping control. The feedback control algorithms were iterated until best simulation results were obtained.

The speed loop was closed, and transient response was tested with both current controller and speed control simulation of the starting mode without load is done followed by step in speed reference Ω_{ref} = 100rad/s, The load torque T_r is applied in t = 0.1s.

The validity of the control is demonstrated through the results shown in Fig. (5, 6, 7, 8) the response for step in speed command is chow in Fig. 5, the speed response is completely robust with perfect rejection of load disturbances.

Fig. 6, show the response of the components i_d and i_q. The stator currents which are controlled by PWM generated by backstepping control, high frequency harmonics that produce high frequency torque ripple.

High accuracy and strong robustness of the nonlinear backstepping are providing by Fig. 7, when speed reversion around (+100 rad/s, -100 rad/s) are applied.

Fig. 8, shows the robustness to parametric variations, where inertia moment J is doubled. There is no overshoot, but the settling time is doubled compared with that of Fig. 6.

6. CONCLUSION

A nonlinear backstepping control method has been proposed and used for the control of a permanent magnet synchronous machine. Simulation Results show good performances obtained with proposed control, with a good choice of parameters of control. The speed control operates with enough stability.

In this approach the components I_d and I_q is regulated using backstepping control, so that I_d is zero, the controller is designed in the total system including switching devices.

The simulation results show, Fast response without overshoot and robust performance to parametric variation and disturbances in all the system

REFERENCES

Chaouch. S. and Naït-Saïd, M. (2006). Backstepping Control Design for Position and Speed Tracking of DC Motors, *Asian Journal of Information Technology 5*.

Ghafarim A. S. (2005). Hybrid stepper motor backstepping control in micro-step operation. *Proceedings of ASME International Mechanical Engineering Congress and Exposition, Orlando, Florida USA.*

Labrique, F. Buyse, H. Seguier, G. and R. Bausière, R. (1998). Converters power electronics command and dynamic behaviour. *Dept. Mines, Sophia-Antipolis*, Paris Univ, France, vol. 5.

Mademlis, C. Agelidis, V. G. (2001). On Considering Magnetic Saturation with Maximum Torque to Current Control in Interior Permanent Magnet Synchronous Motor Drives. *IEEE Trans.* Energy Conversion, Vol. 16, N^o. 3.

Pragasan, P. and R. Krishnan, R. (1988). Modeling of permanent magnet motor drives. *IEEE Trans. Industrial electronics*, vol. 35, no.4.

Rahman, M. A. Vilathgamuwa, D. M. Uddin, M. N. and Tseng, K. J. (2003). Nonlinear Control of Interior Permanent-MagnetSynchronous Motor. *IEEE Trans. Industry applications*, vol. 39, no. 2.

Shun-Sheng, K. and Jung-Shan, L. (2005). Sensorless Speed Tracking Control with Backstepping Design Scheme for Permanent Magnet Synchronous Motors. *Proceedings of IEEE Conference on Control Applications* Toronto, Canada.

Appendix

Three phases PMSM parameters: Rated output power 1500 W, Rated phase voltage 220/380V,
Rs=1.4 Ω, φ=0.154wb, Ld =6.6mH, Lq =5.8mH
F=0.00038N.m.s/rad, J=0.00176, kg.m^2

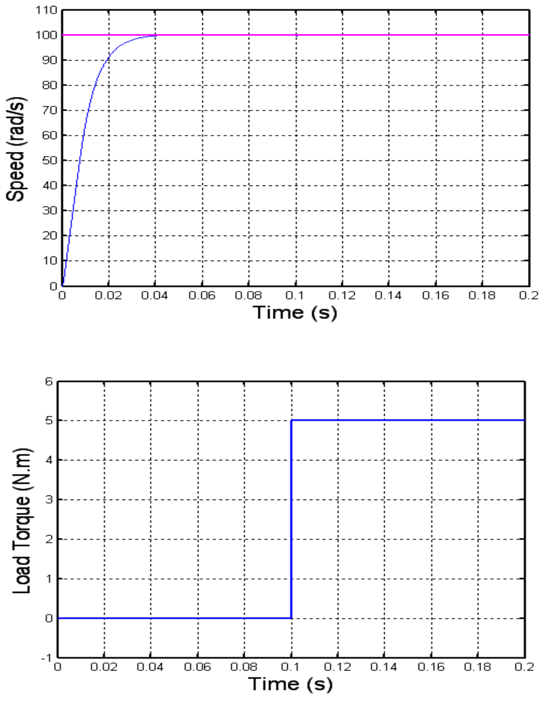

Fig. 5. Response under load disturbance

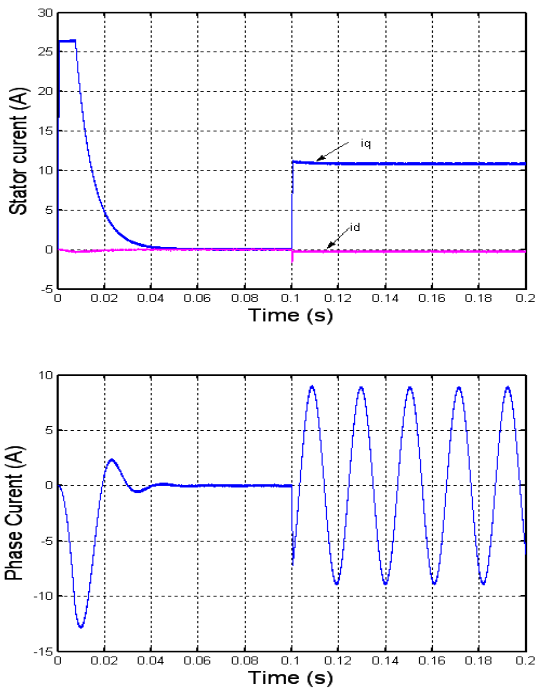

Fig. 6. Stator current under load disturbance

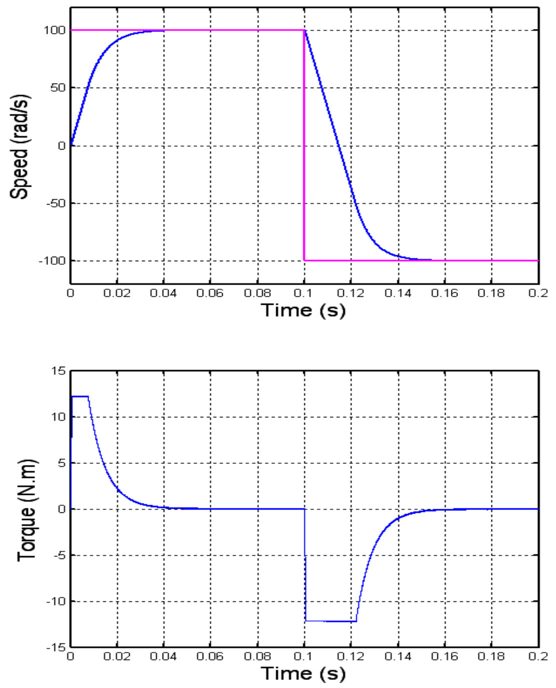

Fig. 7. Speed reversion (+100 rad/s, -100rad/s)

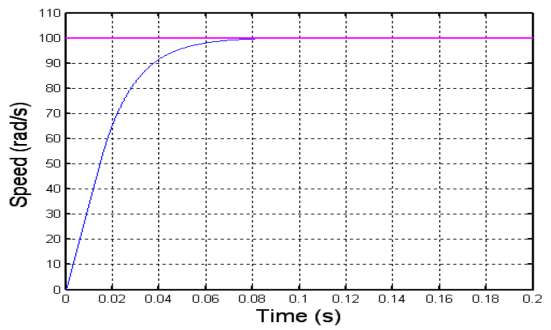

Fig. 8. Response under inertia variation (J=2.Jn)