Advanced Materials Research *Online: 2011-11-29*
ISSN: 1662-8985, Vols. 403-408, pp 4674-4680
doi:10.4028/www.scientific.net/AMR.403-408.4674

A New Approach on Stabilization Control of an Inverted Pendulum, Using PID Controller

Kaveh Razzaghi[1,a] and Ali Akbar Jalali [2,b]

Electrical Engineering Department, Iran University of Science and Technology, Tehran, Iran

[a] k.razzaghi@ee.iust.ac.ir, [b] drjalali@iust.ac.ir

Keywords-PID; MIMO; Inverted Pendulum; Linearization

Abstract—Inverted Pendulum is a standard problem in control systems and is appropriate for depicting linear control principles. In this system there is an inverted pendulum connected to a cart that moves along a horizontal track with the help of a motor. We can determine the cart's position and velocity from the motor and the rail track limits the cart's movement in a bidirectional path. The pendulum's angle of deviation and the position of the cart are determined by two sensors mounted on the system. Essential measurements and motor control signals are generated by a medium control board linking the computer and the system. Analysis of the results and yielding the control commands are done with the help of a MATLAB program. This is indeed a single input-dual output system because we must be able to control two parameters (pendulum's angle and cart's position) with just one control signal to the motor. Since the PID (Proportional Integral Derivative) controller is usually proper for SISO (Single Input Single Output) systems, we are eager to propose a procedure to control one of these parameters underneath the other. In this paper two tactics are described: 1. controlling the cart's position beneath the pendulum's angle, and 2. controlling the pendulum's angle beneath the cart's position. Regarding the results, one method is proven to be superior. We also mention some practical considerations in this paper.

INTRODUCTION

Stabilization control of an inverted pendulum mounted on a moving cart, which moves along a horizontal track, is a classic problem in control systems [1,6]. It is a complicated, nonlinear, unstable, multivariable, and high order system [2]. And because of its intrinsic nonlinearity trait, it's suitable for illustrating ideas in nonlinear control systems. The inverted pendulum system inherently has two equilibrium points, one of which is stable while the other is unstable [1] .These two equilibrium points are proven by Lyapunov's stability theory with different methods [3,4,5]. The stable equilibrium is when the pendulum is pointing downwards and in the absence of any control force the system naturally tends to return to this state. The unstable equilibrium however, is when the pendulum is standing exactly upwards and so a control force is needed to keep it in this position. There are two main considerations in controlling an Inverted Pendulum: one is how to swing up fast from the initial position to operating position; the other is how to stabilize the Inverted Pendulum at the operating position [6]. PID (Proportional Integral Derivative) controllers are the best established class of control systems; however, they cannot directly be used in several more complicated cases, especially if MIMO (Multi Input Multi Output) systems are considered [7].
In this paper two unique strategies are implemented to control the stabilization of the pendulum and the position of the cart simultaneously with the use of PID controller (Controlling the cart's position beneath the pendulum's angle, and controlling the pendulum's angle beneath the cart's position). In this approach the results show that the latter procedure has more advantages than the former one.

MATHEMATICAL MODELING

The defined Inverted Pendulum System is demonstrated in Fig.1. It includes a cart, a pendulum attached to the cart, a rail track to limit the motion to a straight course and also to determine the cart's position. The pendulum is connected to the cart's top surface so it can freely swing in a plane including the rail track and the cart. Since the cart is restricted to move within the track borders, its position is measured relative to the center of the track while the pendulum's deviation angle is measured relative to the unstable equilibrium point. To make the system's equations easier and simple we ignored the effects of friction in the simulations but we noticed them in practice.

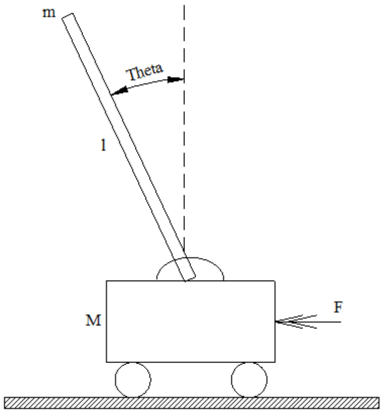


Figure 1. Inverted Pendulum's free diagram

M	Cart's Mass	0.5 kg
m	Pendulum's Mass	0.05 kg
l	Pendulum's Center of Inertia distance	0.3 m
I	Pendulum's Moment Inertia	0.006 kg.m^2
F	Applying force to the cart	
x	Cart's Position	
θ	Pendulum's Angle	

Using the dynamic rules the system's equations are as (1) and a detailed description of deriving the equations can be found in [7].

$$\begin{cases}(M+m)\ddot{x}+ml\ddot{\theta}\cos\theta+ml\dot{\theta}^2\sin\theta=F\\ (I+ml^2)\ddot{\theta}-ml\sin\theta=-ml\ddot{x}\cos\theta\end{cases} \quad (1)$$

Since MATLAB can only work with linear functions, we should linearize the above equations around θ=0, and solve them in Laplace domain in order to reach (2):

$$\begin{cases}\dfrac{\theta(s)}{U(s)}=\dfrac{\frac{ml}{q}s}{s^3+\frac{(M+m)mgl}{q}s+\frac{bmgl}{q}}\\ \dfrac{X(s)}{U(s)}=\dfrac{-\frac{(I+ml^2)}{q}s^2+\frac{mgl}{q}}{s^4+\frac{(M+m)mgl}{q}s^2+\frac{bmgl}{q}s}\\ q=(ml)^2-(M+m)[I+ml^2]\end{cases} \quad (2)$$

In which U is the motivating force generated by motor, and the voltage is the direct control signal exerted to the motor. If we want to simulate the system we need the motor's transfer function as well. To find the motor's transfer function we assume that the selected motor has a shorter transient response than the pendulum's response time. In other words if the motor does not meet this criteria we will not be able to control the pendulum. Therefore, regarding to (3) as equations for the motor's steady state response:

$$\begin{cases}\dfrac{V-k_\omega\omega}{R}=k_i\tau\\ \alpha\dot{\omega}=\tau\\ F=\beta\tau\end{cases} \quad (3)$$

In which V is voltage, ω and τ are motor's angular velocity and torque, K_ω and K_i and R are the motor's velocity, torque and resistance constants, α and β are mechanical constants and finally F is applied force to the cart. By solving (3) in the Laplace domain we'll have the motor's transfer function as (4):

$$\frac{\tau(s)}{V(s)} = \frac{s}{Rk_i s + \frac{k_\omega}{\alpha}} \tag{4}$$

SYSTEM SIMULATION USING PID CONTROLLER

Since the PID controller is able to control only one of the system's parameters simultaneously, in order to control the pendulum's angle and the cart's position at the same time we need two PID controllers in which one of the parameters is taken as the main parameter, and controlled directly by the motors torque, while the other parameter is controlled by applying its effect on the main parameter's reference point. So there could be two scenarios in which either the pendulum's angle, or the cart's position might be taken as the main parameter. We should indicate that as the control factor in both of the above circumstances is the exerted force to the cart, to compensate the motor's effect as the medium between voltage and force we must add the motor's angular velocity multiplied by a constant, relative to K_ω , to the output voltage of the controller. Therefore, in spite of the velocity, the motor's torque and consequently the applied force are relative to the input voltage. In order to find the proper <u>*k*</u> in MATLAB's simulink section we should design a diagram similar to Fig. 2:

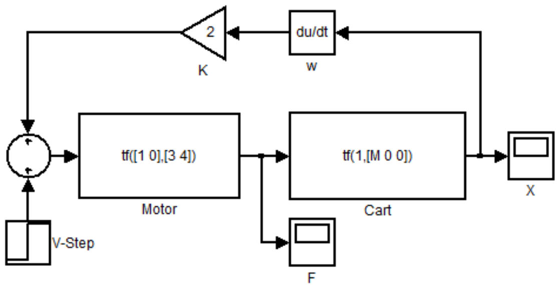
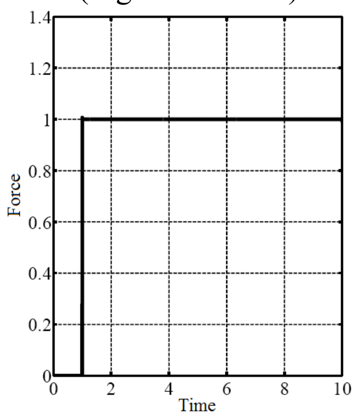
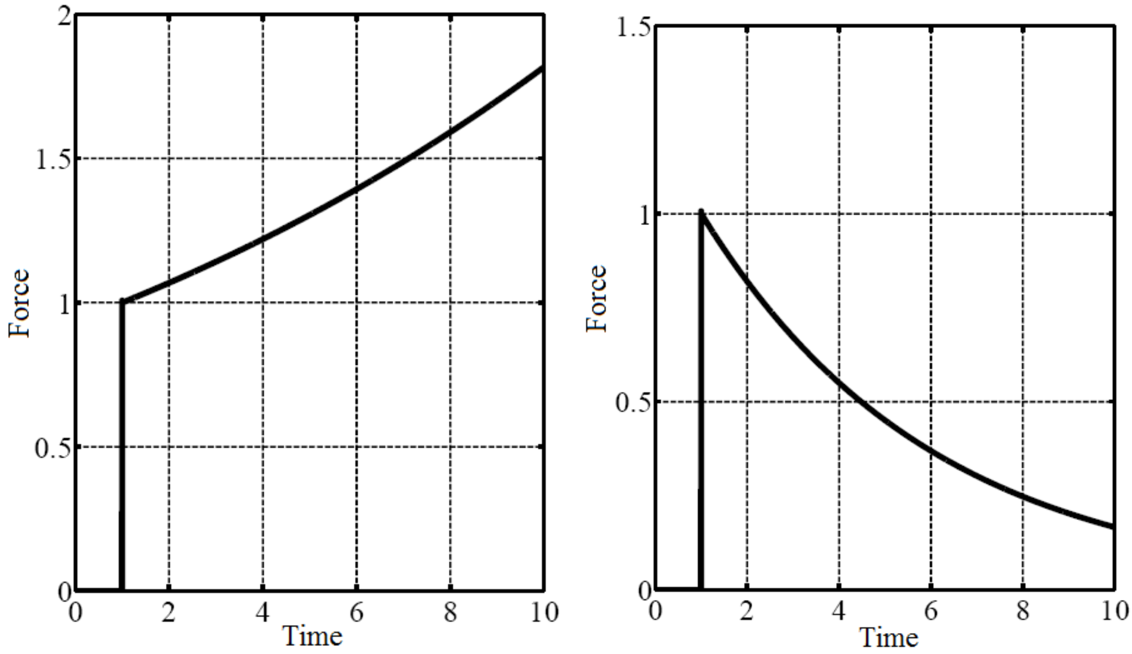


Figure 2. MATLAB simulink diagram for determining the motor's constant (k), in which we have the cart's approximate transfer function and the motor's proposed one.

In this case if <u>*k*</u> equals 2, a constant torque will be generated (Fig. 3.a), otherwise the torque will be either increasing or decreasing by time (Fig. 3.b and 3.c).



(a)



(b) (c)

Figure 3. Motor’s time response to a constant applied voltage in different values of k. (a. k=2, b. k>2, c. k<2)

Controlling The Cart’s Position Beneath The Pendulum’s Deviation Angle. According to the previous statements, in order to control both of the parameters simultaneously we must consider one as the primary variable. If we consider the pendulum’s angle as the main parameter, we must take in the cart’s position in the reference point of pendulum. If the pendulum’s reference point remains constantly θ=0 in spite of the cart’s position and velocity, the pendulum might reach to the equilibrium point while the cart won’t be in control. In this case, the cart may have any possible position and velocity and it might even reach the track borders and have a collision with the endings. To control the cart’s position beneath the pendulum’s angle in simulation the procedure is shown in Fig. 4:

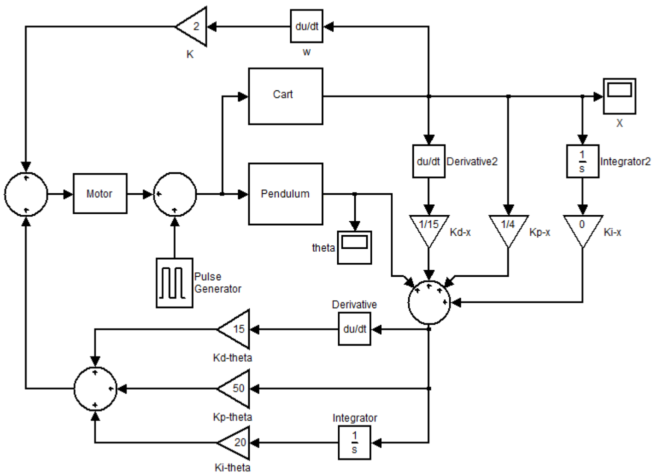
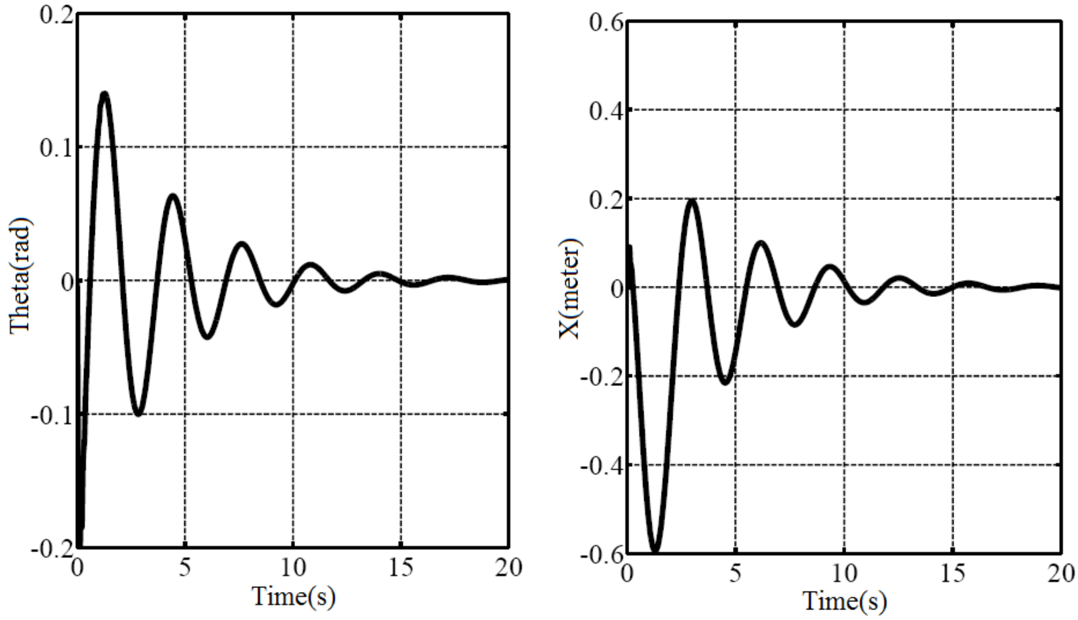


Figure 4. MATLAB simulink diagram in order to control the cart’s position beneath the pendulum’s angle.

So, regarding to Fig. 4 if for example the cart is on the right hand side of its reference point, the pendulum’s reference will be drawn to the left side in order to tend the cart moving left and vice versa. The pendulum’s angle and the cart’s position after the impulsive agitation are illustrated in Fig. 5:



(a) (b)

Figure 5. The time responses of Inverted Pendulum system while the cart's position is controlled beneath the pendulum's angle. a. Time response of pendulum's angle and b. Time response of cart's position.

Controlling The Pendulum's Angle Beneath The Cart's Position. The second approach to control the system is considering the cart's position as the main control variable and somehow taking the pendulum's angle effect on the cart's reference point. To control the pendulum's angle beneath the cart's position in simulation we may design a diagram as Fig. 6.

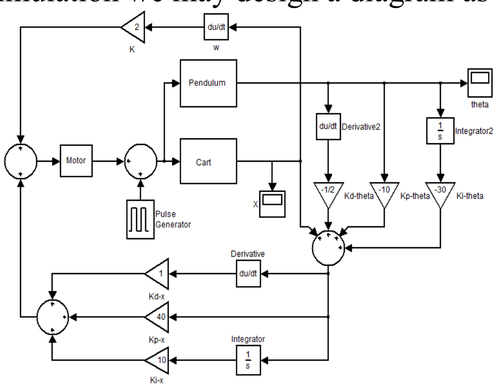


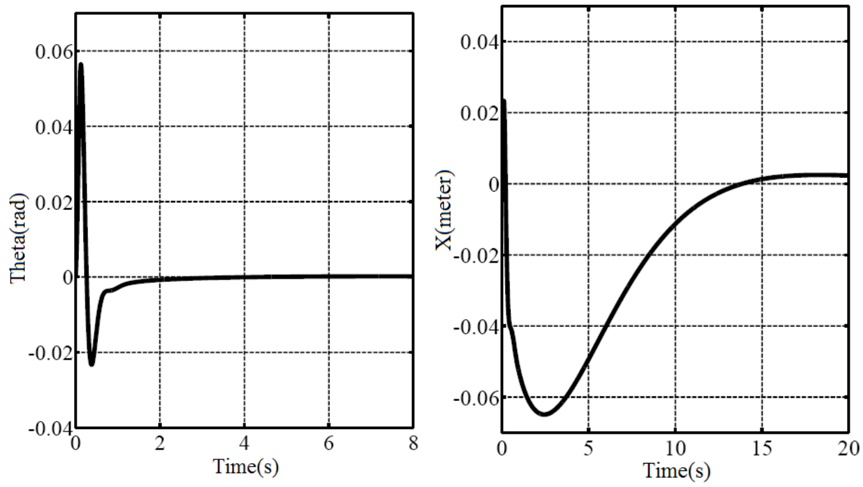
Figure 6. MATLAB simulink diagram in order to control the pendulum's angle beneath cart's position.

With regard to the ignorable mass of pendulum in comparison to the cart's mass, for more simplicity we can use (5) as a transfer function from the cart's position to the applying force:

$$\frac{X(s)}{F(s)} = \frac{1}{Ms^2} \tag{5}$$

In this procedure for example if the pendulum starts moving to the left side because of an agitation, the cart's reference point will be drawn to the left until the pendulum reaches the equilibrium point again, and vice versa. So, the pendulum's angle is controlled indirectly beneath the cart's position.

In this case the simulation results are shown in Fig. 7.a and Fig. 7.b:



(a) (b)

Figure 7. The time responses of Inverted Pendulum system while the pendulum's angle is controlled beneath the cart's position. a. Time response of pendulum's angle. b. Time response of cart's position.

PRACTICAL CONSIDERATIONS

In practical experiment we should notice the following notes:

- Sensors and procedures used to measure the pendulum's angle and cart's position should be as noiseless as possible since even a little noise causes the extra oscillations in pendulum and cart's movement, so the system won't reach the steady state status at all.
- In order to find the motor's velocity constant (as implied previously, it is used for making the generated torque linearly relative to the input voltage) we apply various voltages to the motor and measure its velocity. Then by solving the steady state motor equations the $\underline{k}$ constant yields.
- In practice we must take account of friction instantly in the moving direction.
- If we consider the applying force as (6):

$$F = k_p\theta + k_d\frac{\delta\theta}{\delta t} + k_i\int\theta\,.\,dt \tag{6}$$

The system is linearized around pendulum's equilibrium point. In order to take account of this system's nonlinear trait, we must apply the force F' as shown in the figure (8) so that the pure force exerted to the pendulum is the same as F.

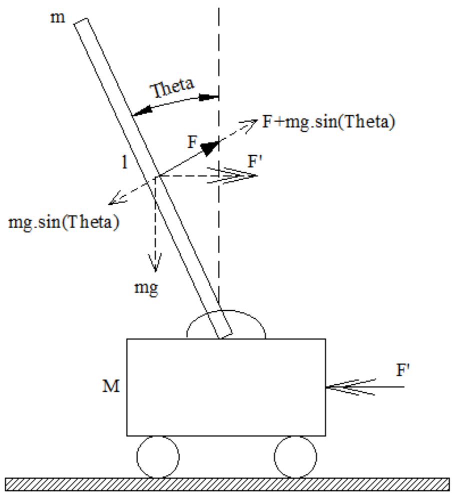


Figure 8. Inverted Pendulum's free body diagram and the applied forces in order to compensate for the nonlinearity trait.

In this case the force F’ generated by motor is shown in (7):

$$F' = F.\sec\theta + mg.\tan\theta \tag{7}$$

CONCLUSIONS

In this paper two unique procedures were proposed and compared for controlling The Inverted Pendulum System with the use of PID controller. For more simplicity in simulation, linear approximations were used in the equations but in practice the nonlinearity trait was compensated by adding several parts to the control signal equations. Considering either pendulum’s angle error or cart’s position error as the main control parameter, two different controllers were designed and implemented.

After comparing the time responses of these two algorithms to a same impulsive agitation, it’s obvious that the second procedure -i.e. controlling the inverted pendulum’s angle beneath the cart’s position, has more advantages: Oscillations and overshoot is less, and the convergence is extremely faster.

Regarding to the above statements, one can conclude that in spite of the fact that a PID controller is not merely appropriate to control multivariable systems, it is possible to use several PID controllers subsequently as though each controller introduces its own controlling variable in the other’s reference point up to the main variable. Therefore, by controlling the primary parameter directly we might be able to control the whole system properly. So, if a system is merely a multivariable, one cannot conclude whether a PID controller is able to take the control of that system or not.

References

[1] S. Mirsaeed Ghazi, Ali A. Jalali, “Low frequencies optimal control of an Inverted Pendulum.”, International Conference on E-Learning and Industrial Electronics (IEEE-ICELIE), pp 85-90, 2006.

[2] Q.R. Li, W.H. Tao, S. Na, C,Y Zhang, L.H Yao, “ Stabilization control of Double Inverted Pendulum System.” , 3rd International Conference on Innovative Computing Information and Control (ICICIC), pp 417- 423, 2008.

[3] J. Aracil, J.A. Acosta, F. Gordillo, ”Kinetic energy shaping in the Inverted Pendulum.”, 6th IFAC Symposium on nonlinear Control Systems (NOLCOS), 2004.

[4] G.D Franklin and J.D Powel. “Digital control of dynamic systems”, Menlo Park, CA: Adison Wesley Longman, 1998.

[5] J.J Slotin and W. Li, Applied nonlinear control, Prentice Hall, USA, pp 105-122, 1998.

[6] J. Xie, X. Xu, K. Xie, “Modeling and simulation of the Inverted Pendulum based on granular hybrid system.”, Chinese Control and Decision Conference (CCDC), pp 3795-3799, 2008.

[7] Regents of the University of Michigan. (1996), “Control tutorials for MATLAB and simulink”, [Online]. Available http://www.engin.umich.edu/class/ctms/examples/pend/invpid.htm.